Math 364
Fall 2008
Exam 2 - Part I

Name:
Score: /100

Show all work and explain your reasoning. Start all problems on the top of the front of a new piece of paper. Work can carry over to the back of the page.

1. **Definitions.** Fill in the remainder of the sentence to complete the definition.
   (a) For two integers, a and b, a divides b if :
   (b) An integer a is odd if:
   (c) Let a and b be natural numbers. The greatest common divisor of a and b is:
   (d) Explain how mathematical induction works to prove a statement for all natural numbers.

2. Give a useful denial of each statement. No explanation is necessary.
   (a) The weather is hot and humid.
   (b) The natural numbers x, y, and z are prime.
   (c) At least one of the natural numbers x, y, and z are prime.
   (d) If a triangle is equilateral, then it is isosceles.
   (e) For every rational number r, the number $\frac{1}{r}$ is rational.
   (f) -2 and -3 are solutions of the equation $x^2 - 5x + 6 = 0$.

3. If possible, give an example of each of the following, with an explanation of how your example satisfies the required conditions:
   (a) a true conditional sentence with a true converse.
   (b) a true conditional sentence with a false converse.
   (c) a true conditional sentence with a false contrapositive.

4. If possible, give an example of each of the following, with an explanation of how your examples satisfies the required conditions:
   (a) a false conditional sentence with a true converse.
   (b) a false conditional sentence with a true inverse and a false converse.
   (c) a true conditional sentence with a false contrapositive.

5. **True/False.** State whether each claim is true or false. If it is false, provide a counterexample or justification.

   (a) For all real numbers y, there exists a real number x so that $x + y = 0$.

   (b) If a and b are positive integers and $a \mid b$ and $b \mid a$ then $a = b$.

   (c) For every prime $p > 5$, $p^2 - 1$ is divisible by 12.

   (d) Let m and n be positive integers, and let q and r be such that $m = nq + r$. Then $gcd(m, n) = gcd(m, r)$.

6. Find $gcd(219, 69)$. Show all work.

7. Find integers x and y so that $219x + 69y = gcd(219, 69)$

   Show all work and explain your reasoning. Start all problems on the top of the front of a new piece of paper. Work can carry over to the back of the page.

8. Prove or disprove: Let m be an integer. If m^2 is even, then m is even.

9. Prove or disprove: The product of a rational number and an irrational number is irrational.

10. Prove or disprove: The sum of two integers is even if and only if they are both even or both odd.

11. Prove or disprove: For all natural numbers n, $7^n - 2^n$ is divisible by 5.

12. Prove or disprove: If a and b are natural numbers, then $gcd(a, b) \cdot lcm(a, b) = ab$.

13. Prove **one** of the following:

    (a) $\sqrt{2}$ is irrational.

    (b) Every natural number $n > 1$ is either prime of the product of primes.