Math 364
Spring, 2004 Name:
Exam 1 - §§1.7 - 2.5

Show all work and explain your reasoning. Answer all questions. Start all problems on the top of the front of a new page of your blue book. Short answer problems can be completed on the same page, and work can carry over to the back of the page.

1. **Definitions.** (8 points) Fill in the remainder of the sentence to complete the definition.
   (a) Let a and b be natural numbers. Then the greatest common divisor of a and b is _____
   (b) For any set, A, the complement of the set, A', is _____
   (c) For a family of sets, $\mathcal{A} = \{A_\alpha \mid \alpha \in \Delta\}$, the intersection, $\bigcap_{\alpha\in\Delta} A_\alpha$, is _____
   (d) For a set A, the power set of A is denoted _____ and is defined to be _____.

2. **True / False.** (24 points) State if the following are true or false. If true, provide a brief proof. If false, provide a counterexample or brief explanation.
   (a) For every set A, $\emptyset \in A$.
   (b) $\{\emptyset\} \subseteq \{\emptyset, \{\emptyset\}\}$
   (c) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
   (d) For any sets A and B, $A \cap B \subseteq A$
   (e) For any set A, $A \cup \emptyset = \emptyset$
   (f) For any sets A and B, $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$

3. (10 points) Let p be prime and let a and b be natural numbers. Prove or disprove the following: $GCD(p, a) = 1 \iff p$ does not divide a.

4. (10 points) Prove or disprove: If $A \subseteq B \cup C$ and $A \cap B = \emptyset$ then $A \subseteq C$.

5. (14 points) The symmetric difference of two sets, A and B is defined by
$$A \Delta B = (A - B) \cup (B - A).$$
Prove that $A \Delta B = B \Delta A$.

6. (12 points) Prove or disprove: Let $\mathcal{A} = \{A_\alpha \mid \alpha \in \Delta\}$ be an indexed family of sets, and let B be a set. Prove that
$$B \cap \bigcup_{\alpha\in\Delta} A_\alpha = \bigcup_{\alpha\in\Delta} (B \cap A_\alpha)$$

7. (12 points) Prove or disprove: For all natural numbers n,
$$6 \,\big|\, n^3 - n$$

8. (10 points) State and prove the Well-Ordering Principle.

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1. Know the following definitions:
   (a) GCD
   (b) LCM
   (c) union (two sets and family of sets)
   (d) intersection (two sets and family of sets)
   (e) set difference
   (f) power set
   (g) complement of a set
   (h) pairwise disjoint
   (i) PMI
   (j) Archimedean principle
   (k) WOP
   (l) Division algorithm
   (m) Euclidean algorithm
2. Be able to prove the following theorems:
   (a) Every natural number $n > 1$ is either prime or is the product of prime factors.
   (b) Well-ordering Principle