
Show all work and explain your reasoning. Answer all questions. Start all problems on the top of the front of a new page of your blue book. Short answer problems can be completed on the same page, and work can carry over to the back of the page.

1. **Definitions.** Fill in the remainder of the sentence to complete the definition.

- (a) For two integers, a and b , a divides b if :
- (b) An integer a is odd if:

2. Write the contrapositive of the following statement. No explanation is necessary.

If xy is odd then both x and y are odd.

3. Give a useful denial of each statement. No explanation is necessary.

- (a) Sue will choose yogurt but will not choose ice cream.
- (b) All squares are rectangles.
- (c) There is a unique even prime integer.
- (d) If xy is odd then both x and y are odd.

4. Give an example of:

- (a) a false conditional sentence with a true converse.
- (b) a true conditional sentence with a false contrapositive.

Explain how your example satisfies the required conditions.

5. **True/False.** State whether each claim is true or false in the universe of real numbers. If it is true, provide justification. If it is false, provide a counterexample.

- (a) There exists a real number x so that $3(2 - x) = 2 + 8(1 - x)$
- (b) For all real numbers x , if $x \geq 0$ then $x^2 + x + 41$ is prime.
- (c) There exists a real number x so that for all y , $x + y = 0$.
- (d) For all real numbers y , there exists a real number x so that $x + y = 0$.

6. Prove one of the following:

- (a) Prove or disprove: For each natural number n , $5^n - 2^n$ is divisible by 3.
- (b) Prove or disprove: If a , b , and c are integers such that a and b are relatively prime and $a \mid bc$ then $a \mid c$.

7. Prove any three of the following four statements:

- (a) For any positive integers, a , b , c , and d , if a divides b and c divides d , then ac divides bd .
- (b) For any integers x , y , and z , if x does not divide yz then x does not divide z .
- (c) If n^2 is an odd integer, then n is an odd integer.
- (d) For any integers x and y , if xy is odd then either x or y is even.