
Show all work and explain your reasoning. Answer all questions. Start all problems on the top of the front of a new page of your blue book. Short answer problems can be completed on the same page, and work can carry over to the back of the page.

1. **Definitions.** (6 points) Fill in the remainder of the sentence to complete the definition.

- (a) For two integers, a and b , a divides b if :
- (b) An integer a is odd if:

2. (6 points) Write the contrapositive of the following statement. No explanation is necessary.

If xy is odd then both x and y are odd.

3. Give a useful denial of each statement. No explanation is necessary.

- (a) (4 points) Sue will choose yogurt but will not choose ice cream.
- (b) (4 points) All squares are rectangles.
- (c) (5 points) There is a unique even prime integer.
- (d) (5 points) If xy is odd then both x and y are odd.

4. (10 points) Give an example of:

- (a) a false conditional sentence with a true converse.
- (b) a true conditional sentence with a false contrapositive.

Explain how your example satisfies the required conditions.

5. (20 points) **True/False.** State whether each claim is true or false in the universe of real numbers. If it is true, provide justification. If it is false, provide a counterexample.

- (a) $(\exists x)(3(2 - x) = 5 + 8(1 - x))$
- (b) $(\forall x)(x \geq 0 \Rightarrow x^2 + x + 41 \text{ is prime})$
- (c) $(\exists x)(\forall y)(x + y = 0)$
- (d) $(\forall y)(\exists x)(x + y = 0)$

6. (10 points) Prove that $\sqrt{2}$ is irrational.

7. (30 points) Prove any three of the following four statements:

- (a) For any positive integers, a , b , c , and d , if a divides b and c divides d , then ac divides bd .
- (b) For any integers x , y , and z , if x does not divide yz then x does not divide z .
- (c) If n^2 is an odd integer, then n is an odd integer.
- (d) For any integers x and y , if xy is odd then either x or y is even.