Math 364 - Set Theory Problems
Fall 2008

Definition 1 *A set is a specified collection of objects.*

Example 1 *The set of integers between* -1 *and* 4*, including the endpoints, is* $\{-1, 0, 1, 2, 3, 4\}$.

Definition 2 *If* A *is a set and* x *is an object that belongs to* A *then* x *is an element of* A*, denoted* $x \in A$*. If* x *is not an object that belongs to* A*, we say that* x *is not an element of* A *(or is not a member of* A*), denoted* $x \notin A$.

60. List all of the elements in the following sets:
    (a) The set of natural numbers strictly less than 6.
    (b) The set of integers whose square is less than 17.
    (c) The set of prime numbers less than 100.
    (d) The set of rational numbers strictly between 0 and 1.

Definition 3 *If* A *and* B *are sets,* B *is a subset of* A*, denoted* $B \subseteq A$ *if every member of* B *is a member of* A.

Definition 4 *Sets* A *and* B *are equal, denoted* $A = B$*, if* $A \subseteq B$ *and* $B \subseteq A$.

61. True or False, and explain your reasoning. $\mathbb{N}$ is the set of natural numbers, $\mathbb{Q}$ is the set of rational numbers, $\mathbb{Z}$ is the set of integers, and $\mathbb{R}$ is the set of real numbers.
    (a) $\mathbb{N} \subseteq \mathbb{Q}$
    (b) $\mathbb{Z} \subseteq \mathbb{N}$
    (c) $\mathbb{Q} \subseteq \mathbb{Z}$
    (d) $\mathbb{N} \subseteq \mathbb{R}$
    (e) $\mathbb{R} \subseteq \mathbb{Q}$
    (f) $(6, 9] \subseteq [6, 10)$
    (g) $[7, 10] \subseteq \mathbb{R}$

Definition 5 *The set with no members is called the empty set and denoted* $\emptyset$.

62. True or False and explain your reasoning.
    (a) $\emptyset \subseteq \mathbb{N}$
    (b) $\emptyset \in \mathbb{N}$

(c) $\emptyset \in \{\emptyset, \{\emptyset\}\}$

(d) $\emptyset \subseteq \{\emptyset, \{\emptyset\}\}$

(e) $\{\emptyset\} \subseteq \{\emptyset, \{\emptyset\}\}$

(f) $\{\emptyset, \{\emptyset\}\} \subseteq \{\{\emptyset, \{\emptyset\}\}\}$

63. True or False and explain your reasoning.

    (a) For every set A, $\emptyset \subseteq A$

    (b) For every set A, $\emptyset \in A$

64. Give an example, if there is one, of sets A, B, and C such that the following are true. If there is no example, state such. Explain your reasoning on all problems.

    (a) $A \subseteq B, B \not\subseteq C$ and $A \subseteq C$.

    (b) $A \subseteq B, B \subseteq C$, and $C \subseteq A$.

    (c) $A \not\subseteq B, B \not\subseteq C$ and $A \subseteq C$.

    (d) $A \subseteq B, B \not\subseteq C$ and $A \not\subseteq C$.

Definition 6 *A subset $A \subseteq B$ is a proper subset if $A \subseteq B$ and $A \neq B$. This is denoted by $A \subset B$.*

65. List all of the subsets of the following sets. Which ones are proper?

    (a) $\emptyset$

    (b) $\{1\}$

    (c) $\{1, 2\}$

    (d) $\{\{\emptyset\}\}$

    (e) $\{\emptyset, \{\emptyset\}\}$

Definition 7 *The power set of a set A is the set of all (proper and not proper) subsets of A. This is denoted $\mathcal{P}(A)$.*

66. Give an example, if there is one, of each of the following. If there is no example, state such. Explain your reasoning on all problems.

    (a) A set A such that $\mathcal{P}(A)$ has 64 elements.

    (b) Sets A and B such that $A \subseteq B$ and $\mathcal{P}(B) \subseteq \mathcal{P}(A)$.

    (c) A set A such that $\mathcal{P}(A) = \emptyset$

    (d) A set A such that $\mathcal{P}(A) = \{\emptyset\}$

    (e) Sets A, B, and C such that $A \subseteq B$, $B \subseteq C$ and $\mathcal{P}(A) \subseteq \mathcal{P}(C)$.

67. True or False and explain your reasoning.

    (a) $\emptyset \in \mathcal{P}(\{\emptyset, \{\emptyset\}\})$
    (b) $\{\emptyset\} \in \mathcal{P}(\{\emptyset, \{\emptyset\}\})$
    (c) $\{\{\emptyset\}\} \in \mathcal{P}(\{\emptyset, \{\emptyset\}\})$
    (d) $\emptyset \subseteq \mathcal{P}(\{\emptyset, \{\emptyset\}\})$
    (e) $\{\emptyset\} \subseteq \mathcal{P}(\{\emptyset, \{\emptyset\}\})$
    (f) $\{\{\emptyset\}\} \subseteq \mathcal{P}(\{\emptyset, \{\emptyset\}\})$

68. True or False and explain your reasoning.

    (a) $3 \in \mathbb{Q}$
    (b) $\{3\} \subseteq \mathcal{P}(\mathbb{Q})$
    (c) $\{3\} \in \mathcal{P}(\mathbb{Q})$
    (d) $\{\{3\}\} \subseteq \mathcal{P}(\mathbb{Q})$
    (e) $\{3\} \subseteq \mathbb{Q}$
    (f) $\{\{3\}\} \in \mathcal{P}(\mathbb{Q})$

69. Prove or disprove: Let n be a natural number and A be a set containing n elements. The number of elements in $\mathcal{P}(A)$ is 2^n.