

Math 364 - Other Miscellaneous Problems  
Fall 2008

95. Prove that  $\sqrt{2}$  is irrational. In other words, prove that if  $r$  is a rational number then  $r^2 \neq \sqrt{2}$ .
96. Let  $m$  be an integer. Prove that if  $m^2$  is odd, then  $m$  is odd.
97. Let  $x$  be an integer. Prove that if  $x^2$  is not divisible by 4 then  $x$  is odd.
98. Let  $x$  and  $y$  be integers. Prove that if  $xy$  is even then either  $x$  is even or  $y$  is even.
99. Let  $x$  and  $y$  be integers. Prove that if  $xy$  is odd then both  $x$  and  $y$  are odd.
100. Let  $x$  be an integer. Prove that if 8 does not divide  $x^2 - 1$  then  $x$  is even.
101. Let  $x$ ,  $y$  and  $z$  be integers. Prove that if  $x$  does not divide  $yz$  then  $x$  does not divide  $z$ .
102. Let  $a$  and  $b$  be integers. Prove that if  $ab$  is odd, then both  $a$  and  $b$  are odd.
103. Let  $a$  and  $b$  be integers. Prove that if  $a - b$  is odd then  $a + b$  is odd.
104. Prove that there exist integers  $m$  and  $n$  so that  $2m + 7n = 1$ .
105. Prove that there do not exist integers  $m$  and  $n$  so that  $2m + 4n = 7$ .
106. Let  $a$ ,  $b$ , and  $c$  be integers. Prove that if  $a$  divides  $b - 1$  and  $a$  divides  $c - 1$  then  $a$  divides  $bc - 1$ .