Math 364 - Chapter 2 HW
Fall 2008

14. (Problem 2.6) Prove and extend *or* extend and salvage: The sum of two odd numbers is an even number.

15. (Problem 2.7) Prove and extend *or* extend and salvage: The product of any three consecutive integers is a multiple of 3.

16. (Problem 2.8) Prove and extend *or* extend and salvage: If the average of four distinct integers is 94, then at least one of the integers must be greater than or equal to 97.

17. (Problem 2.10) Prove or disprove: For any integer $n \geq 1$,

$$1 + 3 + 5 + 7 + \cdots + (2n - 1) = n^2$$

18. (Problem 2.11) Prove or disprove: For any integer $n \geq 1$,

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

19. (Problem 2.12) Prove or disprove:

    (a) For any integer $n \geq 2$,

$$\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right)\left(1 - \frac{1}{16}\right)\cdots\left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$$

    (b) Moreover, we have the formal identity:

$$\prod_{n=2}^{\infty}\left(1 - \frac{1}{n^2}\right) = \frac{1}{2}$$

20. (Problem 2.14) Prove or disprove: Let the sequence of Fibonacci numbers, $\{F_n\}$ be defined by $F_0 = 1$, $F_1 = 1$, and for $n \geq 2, F_n = F_{n-1} + F_{n-2}$. The for all $n \geq 0$,

$$F_{n+2}F_n - (F_{n+1})^2 = (-1)^n .$$