Math 364
Fall 2008 — Name:
Final Exam — Score: /100

Show all work and explain your reasoning. Start all problems on the top of the front of a new piece of paper. Work can carry over to the back of the page.

True/False. State whether each claim is true or false. If it is true, provide a brief justification. If it is false, provide a counterexample.

1. There exists a relation $x \simeq y$ of natural numbers which is reflexive but neither transitive nor symmetric.

2. For every set A, $\emptyset \subseteq A$.

3. There exists a a false conditional sentence with a true inverse.

4. There exists a real number x so that for all y, $x + y = 0$.

5. If a and b are positive integers and $a \mid b$ and $b \mid a$ then $a = b$.

6. Let $X = \{1, 2, 3\}$. Define a relation on elements of $\mathcal{P}(X)$ by $A \simeq B$ iff $A \subseteq B$. This is an equivalence relation on $\mathcal{P}(X)$.

7. The negation of "If a triangle is equilateral then it is isosceles" is "If a triangle is not equilateral then it is not isosceles."

8. Let $A_n = \{1, 2, 3, \ldots, n\}$ and let $\mathcal{A} = \{A_n \mid n \in \mathbb{N}\}$. Then $\displaystyle\bigcup_{A_n \in \mathcal{A}} A_n = \{1, 2, 3, \ldots, n\}$.

9. Let $A_n = \{1, 2, 3, \ldots, n\}$ and let $\mathcal{A} = \{A_n \mid n \in \mathbb{N}\}$. Then $\displaystyle\bigcap_{A_n \in \mathcal{A}} A_n = \{1\}$

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10. Prove or disprove: Let p be a prime integer. If $p \mid a^2$ then $p \mid a$.

11. Prove or disprove: $\sqrt{3}$ is irrational.

12. Prove or disprove: Every natural number $n > 1$ is either prime or the product of primes.

13. Prove or disprove: For all natural numbers n, $n^3 - n$ is divisible by 3.

14. Let $\mathcal{A}$ be a nonempty family of sets and suppose $\emptyset \in \mathcal{A}$. Prove or disprove: $\bigcap_{A \in \mathcal{A}} A = \emptyset$.

15. Prove **one** of the following:

    (a) Prove or disprove: If A and B are sets such that $(A \cup B)' = A' \cup B'$ then $A = B$

    (b) Prove or disprove: If A and B are sets such that $A \cup B = A \cap B$ then $A \cap B' = \emptyset$.