

Math 364 - More set problems  
Fall 2008

**Definition 14** A set of sets is called a family or collection of sets.

**Example 2**

$$\mathcal{A} = \{\{1, 2, 3\}, \{3, 4, 5\}, \{3, 6\}, \{2, 3, 6, 7, 9, 10\}\}$$

is a family consisting of four sets.

107. Consider the family  $\mathcal{A} = \{\{1, 2, 3\}, \{3, 4, 5\}, \{3, 6\}, \{2, 3, 6, 7, 9, 10\}\}$ . Answer the following true/false questions, with reasons.

- (a)  $\{3, 4, 5\} \in \mathcal{A}$
- (b)  $3 \in \mathcal{A}$
- (c)  $\{3, 4, 5\} \subseteq \mathcal{A}$
- (d)  $\{\{3, 4, 5\}\} \subseteq \mathcal{A}$

**Definition 15** Let  $\mathcal{A}$  be an family of sets. The union over  $\mathcal{A}$  is

$$\bigcup_{A \in \mathcal{A}} A = \{x \mid x \in A \text{ for some set } A \in \mathcal{A}\}$$

**Definition 16** Let  $\mathcal{A}$  be an family of sets. The intersection over  $\mathcal{A}$  is

$$\bigcap_{A \in \mathcal{A}} A = \{x \mid x \in A \text{ for every set } A \in \mathcal{A}\}$$

108. Consider the family  $\mathcal{A} = \{\{1, 2, 3\}, \{3, 4, 5\}, \{3, 6\}, \{2, 3, 6, 7, 9, 10\}\}$ . Answer the following, with explanations:

- (a) Find  $\bigcup_{A \in \mathcal{A}} A$
- (b) Find  $\bigcap_{A \in \mathcal{A}} A$

109. Prove or disprove: For every set  $B$  in a family  $\mathcal{A}$  of sets,  $\bigcap_{A \in \mathcal{A}} A \subseteq B$

110. Prove or disprove: For every set  $B$  in a family  $\mathcal{A}$  of sets,  $B \subseteq \bigcup_{A \in \mathcal{A}} A$

111. Prove or disprove: If the family  $\mathcal{A}$  contains at least one set, then  $\bigcap_{A \in \mathcal{A}} A \subseteq \bigcup_{A \in \mathcal{A}} A$ .
112. Let  $A_n = \{1, 2, 3, \dots, n\}$  and let  $\mathcal{A} = \{A_n \mid n \in \mathbb{N}\}$ . Answer the following, with explanations:
- (a) Find  $\bigcup_{A_n \in \mathcal{A}} A_n$
- (b) Find  $\bigcap_{A_n \in \mathcal{A}} A_n$
113. Consider the family  $\mathcal{A} = \{[a, \infty) \mid a \in \mathbb{R}\}$ . Answer the following, with explanations:
- (a) Find  $\bigcup_{A \in \mathcal{A}} A$
- (b) Find  $\bigcap_{A \in \mathcal{A}} A$
114. For each natural number  $n$ , let  $A_n = \left(0, \frac{1}{n}\right)$  and let  $\mathcal{A} = \{A_n \mid n \in \mathbb{N}\}$ . Answer the following, with explanations:
- (a) Find  $\bigcup_{A \in \mathcal{A}} A$
- (b) Find  $\bigcap_{A \in \mathcal{A}} A$
115. For each  $n \in \mathbb{Z}$ , let  $C_n = [n, n + 1)$  and let  $\mathcal{C} = \{C_n \mid n \in \mathbb{Z}\}$ . Answer the following, with explanations:
- (a) Find  $\bigcup_{A \in \mathcal{A}} A$
- (b) Find  $\bigcap_{A \in \mathcal{A}} A$
116. For each  $n \in \mathbb{Z}$ , let  $A_n = (n, n + 1)$  and let  $\mathcal{A} = \{A_n \mid n \in \mathbb{Z}\}$ . Answer the following, with explanations:
- (a) Find  $\bigcup_{A \in \mathcal{A}} A$
- (b) Find  $\bigcap_{A \in \mathcal{A}} A$