

Math 364
Practice Exam 2

Show all work and explain your reasoning. Answer all questions.

1. **Definitions.** Fill in the remainder of the sentence to complete the definition.

(a) For two sets, A and B , the set difference, $A - B$ is defined to be

(b) Other definitions, either from sets or from earlier in the semester.

2. **True / False.** State if the following are true or false. If true, provide a brief proof. If false, provide a counterexample or brief explanation.

(a) For every set A , $\emptyset \subset A$.

(b) $\{\emptyset\} \in \{\emptyset, \{\emptyset\}\}$

(c) $\emptyset \in \{\emptyset, \{\emptyset\}\}$

(d) For any sets A and B , $A \cup B \subseteq A$

(e) For any set A , $A \cap \emptyset = \emptyset$

(f) For any sets A and B , $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$

3. Definition: A family of subsets is pairwise disjoint if the intersection of each pair of subsets is empty.

Let $X = \{1, 2, 3, 4, 5, \dots, 20\}$. Give an example of a family of pairwise disjoint subsets \mathcal{B} of X so that \mathcal{B} has 4 elements and $\bigcup_{B \in \mathcal{B}} B = X$, or explain why such an example cannot exist.

If you provide an example, be sure to explain how it meets the criteria requested.

4. Let a and b be natural numbers.

Prove or disprove: If $GCD(a, b) = 1$ then $LCM(a, b) = ab$.

5. Let A and B be sets with $A \cup B = A \cap B$.

Prove or disprove: $A \cap B' = \emptyset$.

6. Prove or disprove: Every natural number $n > 1$ is either prime or is the product of prime factors.