Math 364
Fall 2008
Practice Exam 1

Name:
Score: /100

Show all work and explain your reasoning. Start all problems on the top of the front of a new piece of paper. All parts of each of the problems 1, 2, 3, and 4 may be done on the same sheet of paper (one sheet for each problem). Work can carry over to the back of the page.

1. **Definitions.** (10 points) Fill in the remainder of the sentence to complete the definition.
   (a) For two integers, a and b, a divides b if :
   (b) An integer a is even if:
2. (3 points each) Give a useful denial of each statement. No explanation is necessary.
   (a) Either milk contains no fat or is green.
   (b) Susan studies but does not get good grades.
   (c) Susan studies and gets help when needed or else she does not get good grades.
   (d) There exists an integer m with the property that for every integer x, there exists an integer y with $xy = m$.
   (e) If interest rates do not go down, the stock market does not go up.
3. (6 points each) If possible, give an example of each of the following, with an explanation of how your examples satisfies the required conditions:
   (a) a false conditional sentence with a true inverse.
   (b) a true conditional sentence with a false contrapositive.
4. **True/False.** (4 points each) State whether each claim is true or false. If it is false, provide a counterexample.
   (a) There exists a real number x so that for all y, $x + y = 0$.
   (b) For all real numbers y, there exists a real number x so that $x + y = 0$.
   (c) If a and b are integers and $a \mid b$ and $b \mid a$ then $a = b$.
5. (15 points) Prove or disprove: $n^3 + n$ is even for every natural number n.
6. (12 points each) Complete the following:
   (a) Prove or disprove: Let a, b and c be integers. If $ac \mid bc$ then $a \mid b$.
   (b) Prove or disprove: Let a, b and c be integers. If $a \mid bc$ then $a \mid b$ or $a \mid c$.