Math 364 - More practice problems
Fall 2008

37. Prove that the sum of the first n natural numbers is $\dfrac{n(n+1)}{2}$.

38. Prove that the sum of the first n powers of 2 is $2^{n+1} - 1$.

39. Prove that for every natural number n,
$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2 .$$

40. Prove that for every natural number n,
$$\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\cdots\left(1 + \frac{1}{n}\right) = n + 1.$$

41. Prove that for every natural number n,
$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}.$$

42. Prove that for every natural number n,
$$1^3 + 3^3 + 5^3 + \cdots + (2n-1)^3 = n^2(2n^2 - 1).$$

43. Let a, b, c, s be integers with $a \neq 0$. Prove each of the following (each counts as a separate problem, by the way):

    (a) If $a \mid b$ and $a \mid c$, then $a \mid (b + c)$ (i.e., the sum of two multiplies of a is a multiple of a.)

    (b) If $a \mid b$ and $a \mid c$ then $a \mid (b - c)$.

    (c) If $a \mid b$ and $b \mid c$ then $a \mid c$.

    (d) If $a \mid b$ and $a \mid c$ then $a^2 \mid bc$.

    (e) If $a \mid b$ and $d \mid c$ then $ad \mid bc$.

44. Let d and n be nonzero integers. Prove or disprove: If $d \mid n^2$ and $d \mid n$.

45. Prove that $3 \mid (n^3 + 2n)$ for every natural number n.

46. Prove that $4 \mid (13^n - 1)$ for every natural number n.