

Math 364 - Chapter 2 HW  
Fall 2008

21. Suppose each of the following three statements is true:  
John is smart.  
John or Mary is ten years old.  
If Mary is ten years old, then John is not smart.  
Which of the following statements are true and why?
- (a) Mary is 10 years old.
  - (b) John is 10 years old.
  - (c) Either John or Mary is not 10 years old.
22. Write a useful negation of each of the following statements:
- (a) All cows eat grass.
  - (b) There is a horse that does not eat grass.
  - (c) There is a car that is blue and weighs less than 4000 pounds.
  - (d) Every math book is either white or hard to read.
  - (e) Some cows are spotted.
  - (f) No car has 15 cylinders.
23. Prove and extend *or* disprove and salvage: The product of two even integers is divisible by 4.
24. Prove and extend *or* disprove and salvage: The square of an odd integer is odd.
- Definition:** For integers  $a$  and  $b$ ,  $a$  divides  $b$  if there exists an integer  $k$  so that  $b = ak$ .
25. Prove or disprove: Let  $a$  and  $b$  be integers. If  $a$  divides  $b$  then  $a$  divides  $bc$  for any integer  $c$ .
26. Prove or disprove: Let  $a$  and  $b$  be integers. If  $a$  and  $b$  are both positive and  $a$  divides  $b$  and  $b$  divides  $a$  then  $a = b$ .
27. Prove or disprove: Let  $a, b$ , and  $c$  be integers. If  $ab$  divides  $c$  then  $a$  divides  $c$ .
28. Prove or disprove: Let  $a, b$ , and  $c$  be integers. If  $ac$  divides  $bc$  then  $a$  divides  $b$ .