

1. Evaluate the following integrals:

(a) $\int \frac{dx}{9 - x^2}$

(b) $\int \frac{dx}{\sqrt{9 - x^2}}$

(c) $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$

(d) $\int \frac{dx}{\sqrt{-2x - x^2}}$

(e) $\int \frac{2 - \cos x + \sin x}{\sin^2 x} dx$

(f) $\int \theta \cos(2\theta + 1) d\theta$

(g) $\int \frac{x^3}{x^2 - 2x + 1} dx$

(h) $\int \frac{dx}{\sqrt{1 + \sqrt{x}}}$

(i) $\int \frac{2 \sin \sqrt{x} dx}{\sqrt{x} \sec \sqrt{x}}$

(j) $\int \frac{\sin 2x dx}{(1 + \cos 2x)^2}$

(k) $\int \frac{dy}{y^2 - 2y + 2}$

(l) $\int \ln \sqrt{x - 1} dx$

(m) $\int \frac{x + 1}{x^2(x^2 + 1)}$

(n) $\int x^3 e^{x^2} dx$

(o) $\int \sin^2 x dx$

(p) $\int \frac{\cos(\sin^{-1} x)}{\sqrt{1 - x^2}}$

(q) $\int \frac{e^t dt}{1 + e^t}$

(r) $\int \frac{\cot v dv}{\ln(\sin v)}$

- (s) $\int (27)^{3\theta+1} d\theta$
 (t) $\int e^x \cos(2x) dx$
 (u) $\frac{dx}{x^2 - 3x + 2}$
 (v) $\int \frac{dx}{(x^2 - 1)^{\frac{3}{2}}}$

2. Determine if the following integrals converge or diverge. Give reasons for your answers. If the integral converges, find its value if possible.

- (a) $\int_0^1 \ln x dx$
 (b) $\int_3^5 \frac{1}{x-4} dx$
 (c) $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$
 (d) $\int_0^\infty \frac{2dx}{x^2-2x}$
 (e) $\int_1^\infty \frac{3x-1}{4x^3-x^2}$
 (f) $\int_0^\infty x^2 e^{-x} dx$
 (g) $\int_1^\infty \frac{e^{-t}}{\sqrt{t}} dt$

3. Calculate the following limits:

- (a) $\lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x}$
 (b) $\lim_{x \rightarrow \infty} x^{\frac{1}{1-x}}$
 (c) $\lim_{t \rightarrow 0} \frac{\tan 3t}{\tan 5t}$

4. Solve the following differential equations

- (a) $\frac{dy}{dx} = -\frac{y \ln y}{1+x^2}$ where $y(0) = e^2$
 (b) $\frac{dy}{dx} + \left(\frac{2}{x+1}\right)y = \frac{x}{x+1}$
 (c) $x \frac{dy}{dx} + 2y = x^2 + 1$
 (d) $xy' + y = x \cos x$

5. Find the lengths of the following curves on the given intervals;

- (a) $x = y^{\frac{2}{3}}, 1 \leq y \leq 8$
 (b) $x = 5 \cos t - \cos 5t, y = 5 \sin t - \sin 5t, 0 \leq t \leq \frac{\pi}{2}$