Math 244
Fall, 2003
Final Exam

This is the final exam given during Fall, 2003. Your exam will cover similar topics, but remember that problems that were presented incorrectly in class are guaranteed to appear. Also, some topics are missing from this review sheet (specifically anything in chapter 13).

Be sure to also review old exams (both versions of exam 2) and quizzes as you study.

1. (3 points) State Fubini's theorem for a rectangular region.
2. (4 points) State the mixed derivative theorem.
3. (3 points) State the definition of the partial derivative of $f(x, y)$ with respect to x.
4. (10 points) If the position vector of a particle is given by

   $$\vec{r}(t) = (t - \sin t)\vec{\imath} + (1 - \cos t)\vec{\jmath}$$

   find all times in the interval $0 \leq t \leq 2\pi$ where the velocity and acceleration vectors are orthogonal.
5. (8 points) A baseball hit by a Boston Red Sox player at a 20° angle from 3 feet above the ground just cleared the left end of the "Green Monster", the left-field wall in Fenway Park. This wall is 37 feet high and 315 feet from home plate. Find the initial speed of the ball and the time that it took the ball to reach the wall.
6. (6 points) Find the volume of the parallelpiped determined by $\vec{u} = 2\vec{\imath} + j$, $\vec{v} = 2\vec{\imath} - \vec{\jmath} + \vec{k}$ and $\vec{w} = \vec{\imath} + 2\vec{k}$.
7. (8 points) Determine the limit

   $$\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4 + y^2}$$

   if it exists. If it does not exist, explain why.
8. (8 points) Use the chain rule to calculate $\frac{\partial z}{\partial u}$ if $z = 4e^x \ln y$ where $x = \ln(u \cos v)$ and $y = u \sin v$.
9. (8 points) Find the equation of the tangent plane to $x^2 - xy - y^2 - z = 0$ at the point $(1, 1, -1)$.
10. (12 points) Find the absolute maxima and absolute minima of $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0$, $y = 2$ and $y = 2x$ in the first quadrant.
11. (9 points) Find the volume of the region enclosed by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 4$.

12. (5 points) Set up, but **do not integrate** the integral to compute the volume of the region that lies inside the sphere $x^2 + y^2 + z^2 = 2$ and outside the cylinder $x^2 + y^2 = 1$.

13. (8 points) Use the Jacobian of the appropriate transformation to prove that

$$dA = dx\ dy = r\ dr\ d\theta$$

14. (8 points) Prove that if $\vec{r}$ is a differentiable vector function of t of constant length then

$$\vec{r}(t) \cdot \frac{d\vec{r}}{dt} = 0$$