Math 244
Fall, 2004 Name: **Solutions**
Quiz #9 - take home part

Complete the following problems. Show all work to receive full credit. This is due at the beginning of class on **Thursday, 7 October**.

1. Find the point in which the line parametrized by $x(t) = 1 - t, y(t) = 3t$, and $z(t) = 1 + t$ intersects the plane $2x - y + 3z = 6$

   Plugging in for x, y, and z:

$$
\begin{aligned}
2x - y + 3z &= 6 \\
2(1-t) - (3t) + 3(1+t) &= 6 \\
-2t + 5 &= 6 \\
t &= -\frac{1}{2}
\end{aligned}
$$

   Plugging back into the parametric equations, we get the point:

$$x = \frac{3}{2}, y = -\frac{3}{2} \text{ and } z = \frac{1}{2}$$

2. Find a parametrization for the line in which the following planes intersect

$$3x - 6y - 2z = 3 \text{ and } 2x + y - 2z = 2$$

$$\vec{n}_1 = 2\vec{\imath} - 6\vec{\jmath} - 2\vec{k} \text{ and } \vec{n}_2 = 2\vec{\imath} + \vec{\jmath} - 2\vec{k}$$

$$
\begin{aligned}
\vec{n}_1 \times \vec{n}_2 &= \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix} \\
&= 14\vec{\imath} + 2\vec{\jmath} + 15\vec{k}
\end{aligned}
$$

   Therefore the direction of the desired line is $\langle 14, 2, 15 \rangle$. The point $(1, 0, 0)$ is on both planes and so the parametric equations for the desired line is:

$$x = 1 + 14t \qquad y = 2t \qquad z = 15t$$

3. Sketch the graph of the function $z = -4x^2 - y^2$. What kind of surface do you get?

   I can't draw this, but it is an elliptic paraboloid.

4. Find the vector equation of the tangent line to the curve

$$\vec{r}(t) = (\sin t)\vec{\imath} + (t^2 - \cos t)\vec{\jmath} + e^t\vec{k} \text{ at } t_0 = 0.$$

$$\vec{v}(t) = (\cos t)\vec{\imath} + (2t + \sin t)\vec{\jmath} + e^t\vec{k}$$
$$\vec{v}(0) = \vec{\imath} + \vec{k}$$
$$\vec{r}(0) = (0, -1, 1)$$

So the direction of the desired line is $\vec{v} = \langle 1, 0, 1\rangle$ and goes through the point $(0, -1, 1)$. So the equation of the line is:

$$t\vec{\imath} + -1\vec{\jmath} + (1 + t)\vec{k}$$

5. Find the equation of the plane through $(1, -1, 3)$ and parallel to the plane $3x + y + z = 7$

The normal is $\langle 3, 1, 1\rangle$, and it goes through the point $(1, -1, 3)$:

$$3(x - 1) + 1(y + 1) + (1)(z - 3) = 0$$
$$3x + y + z = 5$$