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Complete the following problems. Show all work to receive full credit.

1. Let  $L_1$  be the line given by the parametric equations  $x = 3 + 2t, y = -1 + 4t$  and  $z = 2 - t$ . Let  $L_2$  be the line given by the parametric equations  $x = 1 + 4t, y = 1 + 2t$  and  $z = -3 + 4t$ . Are the lines parallel? Do they intersect? Are they skew? Explain your reasoning.

The direction of  $L_1$  is  $\vec{v}_1 = \langle 2, 4, -1 \rangle$  and the direction of  $L_2$  is  $\vec{v}_2 = \langle 4, 2, 4 \rangle$ . Therefore the lines are not parallel. (You can also check this by doing the cross product, which is probably smarter).

The lines do intersect since:

$$\begin{aligned}x_1 &= x_2 \\3 + 2t &= 1 + 4t \\2 &= 2t \\t &= 1\end{aligned}$$

If the  $y$ - and  $z$ -coordinates also match at that time, then the lines intersect at time  $t = 1$ .

$$\begin{aligned}y_1 &= -1 + 4(1) = 3 \text{ and } y_2 = 1 + 2(1) = 3 \\z_1 &= 2 - 1 = 1 \text{ and } z_2 = -3 + 4(1) = 1\end{aligned}$$

Therefore at time  $t = 1$  both lines are at  $(5, 3, 1)$  so the lines intersect.

2. State the name of the type of surface given by the following equations:

(a)  $z^2 + 4y^2 - 4x^2 = 4$

$$-x^2 + y^2 + \frac{z^2}{4} = 1$$

Hyperboloid of one sheet

(b)  $x = -y^2 - z^2$

Elliptic paraboloid

(c)  $9x^2 + 4y^2 + 2z^2 = 36$

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{18} = 1$$

ellipsoid