

Complete the following problems. Show all work to receive full credit.

1. Find  $\frac{dw}{dt}$  for  $w = \ln(x^2 + y^2 + z^2)$  where  $x = \cos t$ ,  $y = \sin t$  and  $z = 4\sqrt{t}$  at  $t = 3$ .

**Two possible approaches:**

$$\begin{aligned}w &= \ln(\cos^2 t + \sin^2 t + 16t) \\ &= \ln(16t + 1) \\ \frac{dw}{dt} &= \frac{16}{16t + 1} \\ \left. \frac{dw}{dt} \right|_{t=3} &= \frac{16}{16(3) + 1} = \frac{16}{49}\end{aligned}$$

**Alternate approach**

$$\begin{aligned}\frac{dw}{dt} &= \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{dz}{dt} \\ &= \frac{2x}{x^2 + y^2 + z^2} (-\sin t) + \frac{2y}{x^2 + y^2 + z^2} (\cos t) + \frac{2z}{x^2 + y^2 + z^2} \left( \frac{2}{\sqrt{t}} \right) \\ x(3) &= \cos 3 & y(3) &= \sin 3 & z(3) &= 4\sqrt{3} \\ x^2 + y^2 + z^2 &= 16(3) + 1 = 49 \\ \frac{dw}{dt} &= \frac{8\sqrt{3} \cdot 2}{49\sqrt{3}} = \frac{16}{49}\end{aligned}$$

2. Find  $\frac{\partial z}{\partial u}$  for  $z = 4e^x \ln y$  where  $x = \ln(u \cos v)$  and  $y = u \sin v$  at  $(u, v) = (2, \frac{\pi}{4})$

$$\begin{aligned}z &= 4e^{\ln(u \cos v)} \ln(u \sin v) \\ &= 4u \cos v \ln(u \sin v) \\ \frac{\partial z}{\partial u} &= 4 \cos v \ln(u \sin v) + 4 \frac{\sin v}{u \sin v} \cdot u \cos v \\ &= 4 \cos v \ln(u \sin v) + \cos v \\ \left. \frac{\partial z}{\partial u} \right|_{(2, \frac{\pi}{4})} &= 4 \cos \frac{\pi}{4} \left( \ln(2 \sin \frac{\pi}{4}) + 1 \right) \\ &= 2\sqrt{2} (\ln \sqrt{2} + 1)\end{aligned}$$