Math 244
Fall, 2004

The following problems are the homework problems for the course. Problems marked with a **(C)** are computational problems and are not eligible for presentation during class. However, the computational problems may appear on quizzes and exams, and are assigned to assure that you have mastered the basic skills required in each section, in addition to having mastered the theory of the topics discussed.

Remember that you are not to consult with other students except on computational problems or problems already presented in class. You can use the textbook for reference, but all solutions to assigned problems should be in your own words, not parroted from the book or other sources.

You can use previous problems on later problems, if they are helpful. However, you cannot use a later problem for help. Part of the challenge of these problems is to find and rectify any typographical errors.

1. (Katie/Amanda) Find the unit vectors that are parallel to the vector $2\vec{\imath} - 3\vec{\jmath}$.
2. (Courtney) Find a vector of length 2 whose direction is the opposite of the direction of the vector $-\vec{\imath} + 2\vec{\jmath}$.
3. (a) (Sarah) Find the unit vectors that are tangent and normal to the curve $x^2 + 2y^2 = 6$ at the point $(2, 1)$.
   (b) (Amanda) If $\vec{v} = \langle v_1, v_2 \rangle$ is a tangent vector, is the vector $\langle -v_2, v_1 \rangle$ always normal?
4. (Ashley) Find the unit vectors that are tangent and normal to the curve $x^2 - 6xy + 8y^2 - 2x - 1 = 0$ at the point $(1, 1)$.
5. (Eunice) Find the unit vectors that are tangent and normal to the curve $\int_e^x \ln(\ln t)dt$ at the point $(e, 0)$.
6. (Clint) An airplane is flying in the direction 25° west of north at 800 km per hour. Find the component form of the velocity of the airplane, assuming that the positive $x-$axis represents due east and the positive $y-$axis represents due north.
7. (Cherie) A bird flies from its nest 5 km in the direction of 60° north of east, where it stops to rest on a tree. It then flies 10 km in the direction due southeast and lands atop a telephone pole. Place and $xy-$coordinate system so that the origin lands on the bird's nest, the $x-$axis points east and the $y-$axis points north.
   (a) At what point is the tree located?
   (b) At what point is the telephone pole located?
8. **(C)** For the vectors $\vec{v} = 2\vec{\imath} + 10\vec{\jmath}$ and $\vec{u} = 2\vec{\imath} + 2\vec{\jmath}$, find:
   (a) $\vec{v} \cdot \vec{u}$

(b) $|\vec{v}|$
(c) $|\vec{u}|$
(d) the cosine of the angle between $\vec{v}$ and $\vec{u}$
(e) the scalar component of $\vec{u}$ in the direction $\vec{v}$
(f) the vector projection, $proj_v\vec{u}$

9. **(C)** For the vectors $\vec{v} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$ and $\vec{u} = \left\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$, find:

   (a) $\vec{v} \cdot \vec{u}$
   (b) $|\vec{v}|$
   (c) $|\vec{u}|$
   (d) the cosine of the angle between $\vec{v}$ and $\vec{u}$
   (e) the scalar component of $\vec{u}$ in the direction $\vec{v}$
   (f) the vector projection, $proj_v\vec{u}$

10. **(C)** Find the measures of the angles between the diagonals of the rectangle whose vertices are $A = (1, 0)$, $B = (0, 3)$, $C = (3, 4)$ and $D = (4, 1)$.

11. (Quiz problem) Suppose that AB is the diameter of a circle with center O and that C is a point on one of the two arcs joining A and B. Show that $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are orthogonal.

12. (Leslie and Patrick) Show that the diagonals of a rhombus (parallelogram with sides of equal length) are perpendicular.

13. (Sarah) Show that squares are the only rectangles with perpendicular diagonals.

14. (Amanda and Ashley) Prove that a parallelogram is a rectangle if and only if its diagonals are equal in length.

15. (Leslie) (Cauchy-Schwartz Inequality) Use the fact that $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$ to show that the inequality $|\vec{u} \cdot \vec{v}| \leq |\vec{u}||\vec{v}|$ holds for all vectors $\vec{u}$ and $\vec{v}$.

16. (Courtney) Show that $\vec{v} = a\vec{\imath} + b\vec{\jmath}$ is perpendicular to the line $ax + by = c$.

17. **(C)** Let $\vec{r}(t) = (2\ln(t+1))\vec{\imath} + (t^2)\vec{\jmath}$ be the position vector of a particle in the plane at time t.

   (a) Sketch a graph of the particle
   (b) Find the velocity vector.
   (c) Find the acceleration vector.
   (d) Find the particle's speed and direction of motion at $t = 1$.
   (e) Write the particle's velocity at $t = 1$ as the product of its speed and direction.

18. (Robert) Let the position vector of a particle in the plane at time t be given by $r(t) = (\sin t)\vec{\imath} + t\vec{\jmath}$ for $t \geq 0$. Find the time(s) when the velocity and acceleration vectors are perpendicular.

19. (Troy) Let the position vector of a particle in the plane at time t be given by $r(t) = (3t + 1)\vec{\imath} + (t^2)\vec{\jmath}$. Find the angle between the velocity and acceleration vectors at $t = 0$.

20. (Amanda) A particle moves in the plane so that its velocity and position vectors are always orthogonal. Show that the particle moves in a circle centered at the origin.

21. (Amanda) A particle moves around the unit circle in the $xy-$plane. Its position at time t is $\vec{r} = x\vec{\imath} + y\vec{\jmath}$, where x and y are differentiable functions of t. Find $\frac{dy}{dt}$ if $\vec{v} \cdot \vec{\imath} = y$. Is the motion clockwise or counterclockwise?

22. **(C)** Find $\lim\limits_{t \to 3} \left(t\vec{\imath} + \dfrac{t^2 - 9}{t^2 + 3t}\vec{\jmath} \right)$. Is the function continuous at $t = 3$?

23. **(C)** Find the equation of the tangent and normal lines to the curve

$$\vec{r}(t) = (2\cos t - 3)\vec{\imath} + (3\sin t + 1)\vec{\jmath} \text{ at } t = \frac{\pi}{4}$$

24. **(C)** Find $\displaystyle\int_1^2 \Big((6 - 6t\vec{\imath} + 3\sqrt{t}\vec{\jmath} \Big)\, dt.$

25. **(C)** Find $\displaystyle\int \left(\frac{1}{t}\vec{\imath} + \frac{1}{5 - t}\vec{\jmath} \right) dt$

26. **(C)** Solve the initial value problem for $\vec{r}$ as a vector function of t given that

$$\frac{d^2\vec{r}}{dt^2} = -\vec{\imath} - \vec{\jmath} \qquad \vec{r}(0) = 10\vec{\imath} + 10\vec{\jmath} \qquad \left.\frac{d\vec{r}}{dt}\right|_{t=0} = \vec{0}$$

27. (The class) The position of a particle in the plane at time t is given by $\vec{r}(t) = (1 - \cos t)\vec{\imath} - (t - \sin t)\vec{\jmath}$. Find the distance the particle travels along the path from $t = 0$ to $t = \frac{2\pi}{3}$.

28. (Riley) At time $t = 0$ a particle is located at the point $(1, 2)$. It travels in a straight line to the point $(4, 1)$, has speed 2 at $(1, 2)$ and constant acceleration $3\vec{\imath} - \vec{\jmath}$. Find an equation for the position vector $\vec{r}(t)$ of the particle at time t.

29. (Patrick, except a) The position of a kite is given by $\vec{r}(t) = \frac{t}{8}\vec{\imath} - \frac{3}{64}t(t - 160)\vec{\jmath}$ where $t \geq 0$ is measured in seconds and distance is measured in meters.
    (a) (Courtney) How long is the kite above ground?
    (b) How high is the kite at $t = 40$ seconds?
    (c) At what rate is the kite's altitude increasing at $t = 40$ seconds?
    (d) At what time does the kite start to lose altitude?

30. (Amanda) Let $\vec{v}$ be a differentiable vector function of t. Show that if $\vec{v} \cdot \left(\dfrac{d\vec{v}}{dt} \right) = \vec{0}$ for all t, then $|\ \vec{v}\ |$ is constant.

31. (Emily) Prove that if $\vec{u}$ is the vector function with constant value $\vec{C}$ then $\dfrac{d\vec{u}}{dt} = \vec{0}$.

32. (Leslie) Show that if $\vec{r}(t) = f(t)\vec{\imath} + g(t)\vec{\jmath}$ is differentiable at $t = c$ then $\vec{r}$ is continuous at c.

33. (Cherie) A projectile is fired at a speed of 840 m/sec at an angle of 60°. How long will it take to get 21 km downrange.

34. (Robert) A baseball is thrown from the stand 32 feet above the field at an angle of 30° up from the horizontal. When and how far away will the ball strike the ground if its initial speed is 32 ft/sec?

35. (Clint) A human cannonball is to be fired with an initial speed of $v_0 = \frac{80\sqrt{10}}{2}$ feet per second. The circus performer hopes to and on a special cushion located 200 feet downrange at the same height as the muzzle of the cannon. The circus is being held in a large room with a flat ceiling 75 feet higher than the muzzle. Can the performer be fired to the cushion without striking the ceiling? if so, what should the cannon's angle of elevation be?

36. (Riley and Amanda) A baseball hit by a Boston Red Sox player at a 20° angel from 3 feet above the ground just cleared the left end of the "Green Monster," the left-field wall in Fenway Park. This wall is 37 feet high and 315 feet from home plate.
    (a) What was the initial speed of the ball?
    (b) How long did it take the ball to reach the wall?

37. (Ashley) In Moscow in 1987, Natalya Lisouskaya set a women's world record by putting an 8 pound 13 ounce shot 73 feet 10 inches. Assuming that she launched the shot at a 40° angle to the horizontal from 6.5 feet above the ground, what was the shot's initial speed?

38. **(C)** Be able to do problems 1-48 in section 9.5

39. (Leslie) Explain why every vertical line in the plane has a polar equation of the form $r = a \sec \theta$

40. (August) Find an analogous polar equation for horizontal lines.

41. (Courtney) Find a formula for the distance between points (r_1, θ_1) and (r_2, θ_2). Be able to explain why your formula works.

42. Find the maximum height above the $x-$axis of the cardioid $r = 2(1 + \cos \theta)$.

43. **(C)** Find the slope of the curve $r = 2 - 3 \sin \theta$ at $(2, \pi)$.

44. **(C)** Find the tangent lines to $r = 2 \sin 2\theta$ at the pole for $0 \leq \theta \leq 2\pi$.

45. **(C)** Find equations for the horizontal and vertical tangents to the curve $r = 3 - 4 \cos \theta$ for $0 \leq \theta \leq 2\pi$.

46. **(C)** Find the area of the region inside the cardioid $r = a(1 + \cos \theta)$ for $a > 0$.

47. **(C)** Find the area inside the six-leaved rose $r^2 = 2 \sin 3\theta$.

48. **(C)** Find the area shared by the circle $r = 2$ and the cardioid $r = 2(1 - \cos \theta)$.

49. **(C)** Find the area inside the circle $r = 6$ above the line $r = 3 \csc \theta$.

50. **(C)** Find the length of the spiral $r = \dfrac{e^\theta}{\sqrt{2}}$ for $0 \leq \theta \leq \pi$.

51. (Joel) Recall the definition of average value of a function from Calc. II. Find the average value of $r = a(1 - \cos \theta)$ for $a > 0$.

51.5. (Amanda) Recall that for parametric equations, arc length is $L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$ for $\alpha \leq t \leq \beta$. Use this to show that the length of the curve in a polar graph is $L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$

52. (Ashley) What can be said about the relative lengths of the curves $r = f(\theta)$ and $r = 2f(\theta)$ where $\alpha \leq \theta \leq \beta$?

53. **(C)** Be able to do problems 1-40 in Section 10.1.

54. **(C)** Find a vector of magnitude 7 in the direction of $\vec{v} = 12\vec{\imath} - 5\vec{k}$.

55. **(C)** Consider the points $P_1(3,4,5)$ and $P_2(2,3,4)$.
    (a) Find the distance between P_1 and P_2.
    (b) Find the direction of $\overrightarrow{P_1P_2}$
    (c) Find the midpoint of the line segment P_1P_2.

56. **(C)** Find the center and the radius of the sphere given by $3x^2 + 3y^2 + 3z^2 + 2y - 2z = 9$.

57. (Emily) Let $ABCD$ be a general, not necessarily planar, quadrilateral in space. Show that the two segments joining the midpoints of the opposite sides of $ABCD$ bisect each other.

58. (Riley) Suppose that A, B, and C are vertices of a triangle and that a, b and c, respectively, are the midpoints of the opposite sides. Show that $\overrightarrow{Aa} + \overrightarrow{Bb} + \overrightarrow{Cc} = 0$.

59. **(C)** For the vectors $\vec{v} = \vec{\imath} + \vec{\jmath}$ and $\vec{u} = \sqrt{2}\vec{\imath} + \sqrt{3}\vec{\jmath} + 2\vec{k}$,
    (a) Find $\vec{v} \cdot \vec{u}$
    (b) $|\,\vec{v}\,|$
    (c) $|\,\vec{u}\,|$
    (d) the cosine of the angle between $\vec{v}$ and $\vec{u}$
    (e) the scalar component of $\vec{u}$ in the direction $\vec{v}$
    (f) the vector projection, $proj_v\vec{u}$

60. **(C)** Write $\vec{u} = 8\vec{\imath} + 4\vec{\jmath} - 12\vec{k}$ as a the sum of a vector parallel to $\vec{v} = \vec{\imath} + 2\vec{\jmath} - \vec{k}$ and a vector orthogonal to $\vec{v}$.

61. **(C)** Find the length and direction of $\vec{u} \times \vec{v}$ and $\vec{v} \times \vec{u}$ for $u = -8\vec{\imath} - 2\vec{\jmath} - 4\vec{k}$ and $\vec{v} = 2\vec{\imath} + 2\vec{\jmath} + \vec{k}$.

62. **(C)** Find the area of the triangle determined by $P(-2,2,0)$, $Q(0,1,-1)$ and $R(-1,2,-2)$.

63. **(C)** Find the volume of the parallelpiped determined by $\vec{u} = \vec{\imath} - \vec{\jmath} + \vec{k}$, $\vec{v} = 2\vec{\imath} + \vec{\jmath} - 2\vec{k}$ and $\vec{w} = -\vec{\imath} + 2\vec{\jmath} - \vec{k}$.

64. (Eunice) Let $\vec{u} = \vec{\imath} + 2\vec{\jmath} - \vec{k}$, $\vec{v} = -\vec{\imath} + \vec{\jmath} + \vec{k}$, $\vec{w} = \vec{\imath} + \vec{k}$, and $\vec{r} = -\frac{\pi}{2}\vec{\imath} - \pi\vec{\jmath} + \frac{\pi}{2}\vec{k}$. Which vectors are perpendicular? parallel?

65. (August a-d, Sarah f-h, Katie i-l, Leslie m-p)Which of the following are always true and which are not always true?

   (a) $\mid \vec{u} \mid = \sqrt{\vec{u} \cdot \vec{u}}$
   (b) $\vec{u} \cdot \vec{u} = \mid \vec{u} \mid$
   (c) $\vec{u} \times \vec{0} = \vec{0} \times \vec{u} = \vec{0}$
   (d) $\vec{u} \times (-\vec{u}) = \vec{0}$
   (e) $\vec{u} \times \vec{v} = \vec{v} \times \vec{u}$
   (f) $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$
   (g) $(u \times \vec{v}) \cdot \vec{v}) = 0$
   (h) $(\vec{u} \times v) \cdot \vec{w} = \vec{u} \cdot (\vec{v} \times \vec{w})$
   (i) $\vec{u} \cdot v = \vec{v} \cdot \vec{u}$
   (j) $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$
   (k) $(-u) \times \vec{v} = -(\vec{u} \times \vec{v})$
   (l) $(c\vec{u}) \cdot \vec{v} = \vec{u} \cdot (c\vec{v}) = c(\vec{u} \cdot \vec{v})$
   (m) $(c\vec{u}) \times \vec{v} = \vec{u} \times (c\vec{v}) = c(\vec{u} \times \vec{v})$
   (n) $\vec{u} \cdot \vec{u} = \mid \vec{u} \mid^2$
   (o) $(\vec{u} \times \vec{u}) \cdot \vec{u} = 0$
   (p) $\vec{u} \times \vec{v}) \cdot u = \vec{v} \cdot (\vec{u} \times \vec{v})$

66. (Clint) Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be vectors. Which of the following make sense, and which do not?

   (a) $(\vec{u} \times \vec{v}) \cdot \vec{w}$
   (b) $\vec{u} \times (\vec{v} \cdot \vec{w})$
   (c) $\vec{u} \times (\vec{v} \times \vec{w})$
   (d) $\vec{u} \cdot (\vec{v} \cdot \vec{w})$

Exam 1 covers to this point

Exam 2 begins here

67. **(C)** Find the vector and parametric equations for the line through $P(-2,0,3)$ and $Q(3,5,-2)$.

68. **(C)** Find the vector and parametric equations for the line through $(1,1,1)$ parallel to the $z-$axis.

69. **(C)** Find the line through $(2,3,0)$ perpendicular to the vectors $\vec{u} = \vec{\imath} + 2\vec{\jmath} + 3\vec{k}$ and $\vec{v} = 3\vec{\imath} + 4\vec{\jmath} + 5\vec{k}$.

70. **(C)** Find the equation of the plane through $(0,2,-1)$ normal to $\vec{n} = 3\vec{\imath} - 2\vec{\jmath} - \vec{k}$.

71. **(C)** Find the equation of the plane through $(1,-1,3)$ parallel to the plane $3x + y + z = 7$.

72. **(C)** Find the equation of the plane through $(1,1,-1)$, $(2,0,2)$, and $(0,-2,1)$.

73. **(C)** Find the distance from the line $x = 2 + t$, $y = 1 + t$, $z = -\frac{1}{2} - \frac{1}{2}t$ to the plane $x + 2y + 6z = 10$.

74. (Courtney) If $\vec{n}_1$ and $\vec{n}_2$ are the normals to two planes, show that the angles between the planes is

$$\theta = \cos^{-1}\left(\frac{\vec{n}_1 \cdot \vec{n}_2}{|\,\vec{n}_1\,||\,\vec{n}_2\,|}\right)$$

75. (Amanda) Find equations for the line in the plane $z = 3$ that makes an angle of $\frac{\pi}{6}$ radians with $\vec{\imath}$ and an angle of $\frac{\pi}{3}$ radians with $\vec{\jmath}$.

76. (Leslie) How can you tell when two planes $A_1x + B_1y + C_1z = D_1$ and $A_2x + B_2y + C_2z = D_2$ are parallel? perpendicular?

77. (Courtney) Suppose that L_1 and L_2 are disjoint nonparallel lines. Is it possible for a nonzero vector to be perpendicular to both L_1 and L_2?

78. **(C)** Be able to do problems 1-12 in section 10.4.

78.5 (Leslie and Ashley) Present a mini-lecture on section 10.4 - quadric surfaces.

79. **(C)** Be able to do problems 1-24 in section 10.5.

80. (Amanda) Find the parametric equations for the line that is tangent to the curve $\vec{r}(t) = (a\sin t)\vec{\imath} + (a\cos t)\vec{\jmath} + bt\vec{k}$ at $t = 2\pi$.

81. (Amanda) At time $t = 0$ a particle is located at the point $(1,2,3)$. It travels in a straight line to the point $(4,1,4)$, has speed 2 at $(1,2,3)$ and constant acceleration $3\vec{\imath} - \vec{\jmath} + \vec{k}$. Find an equation for the position vector $\vec{r}(t)$ of the particle at time t.

82. (Amanda) Show that the vector-valued function

$$\vec{r}(t) = (2\vec{\imath} + 2\vec{\jmath} + \vec{k}) + (\cos t)\left(\frac{1}{\sqrt{2}}\vec{\imath} - \frac{1}{\sqrt{2}}\vec{\jmath}\right) + (\sin t)\left(\frac{1}{\sqrt{3}}\vec{\imath} + \frac{1}{\sqrt{3}}\vec{\jmath} + \frac{1}{\sqrt{3}}\vec{k}\right)$$

describes the motion of a particle moving on the circle of radius 1 centered at the point $(2,2,1)$ and lying in the plane $x + y - 2z = 2$.

83. (Emily) Let $\vec{v}$ be a differentiable vector function of t. Show that if $\vec{v} \cdot \left(\frac{d\vec{v}}{dt}\right) = 0$ for all t, then $|\ \vec{v}\ |$ is constant.

84. (Courtney) Show that the vector function $\vec{r}$ defined by the rule $\vec{r}(t) = f(t)\vec{\imath} + g(t)\vec{\jmath} + h(t)\vec{k}$ is continuous at $t = t_0$ if and only if f, g and h are continuous at t_0.

85. **(C)** Find the length of the curve $\vec{r}(t) = (2\cos t)\vec{\imath} + (2\sin t)\vec{\jmath} + \sqrt{5}t\vec{k}$ for $0 \leq t \leq \pi$.

86. (Cherie) Find the unit tangent vector to the curve $\vec{r}(t) = (2\cos t)\vec{\imath} + (2\sin t)\vec{\jmath} + \sqrt{5}t\vec{k}$ at any time t.

87. **(C)** Find the length of the curve $\vec{r}(t) = (t\cos t)\vec{\imath} + (t\sin t)\vec{\jmath} + \left(\frac{2\sqrt{2}}{3}\right)t^{\frac{3}{2}}\vec{k}$ for $0 \leq t \leq \pi$.

88. (Riley) Find the unit tangent vector to the curve $\vec{r}(t) = (t\cos t)\vec{\imath} + (t\sin t)\vec{\jmath} + \left(\frac{2\sqrt{2}}{3}\right)t^{\frac{3}{2}}\vec{k}$ at any time t.

89. (Leslie) Find the point on the curve $\vec{r}(t) = (5\sin t)\vec{\imath} + (5\cos t)\vec{\jmath} + 12t\vec{k}$ at a distance of 26π units along the curve from the point $(0, 5, 0)$ when $t = 0$ in the direction of increasing arc length.

90. **(C)** Find T, N, and κ for the plane curve $\vec{r}(t) = t\vec{\imath} + (\ln\cos t)\vec{\jmath}$ for $-\frac{\pi}{2} < t < \frac{\pi}{2}$.

91. **(C)** Find the length of the curve $\vec{r}(t) = (\sqrt{2}t)\vec{\imath} + (\sqrt{2}t)\vec{\jmath} + (1-t^2)\vec{k}$ from $(0, 0, 1)$ to $(\sqrt{2}, \sqrt{2}, 0)$.

92. (Robert and Leslie) Find an equation for the circle of curvature of the curve $\vec{r}(t) = t\vec{\imath} + \sin t\vec{\jmath}$ at the point $(\frac{\pi}{2}, 1)$.

93. (Sarah) Explain what T, N, and B measure. Why are they important?

93.5 (Amanda) What does it mean for a curve to be arc length parametrized? How can you tell if a curve is arc length parametrized? How can you make a curve arc length parametrized?

94. (Eunice) Explain what κ and τ measure. Why are they important?

95. **(C)** Find T, N, B, κ, and τ for the space curve

$$\vec{r}(t) = (3\sin t)\vec{\imath} + (3\cos t)\vec{\jmath} + 4t\vec{k}$$

96. **(C)** Find T, N, B, κ, and τ for the space curve

$$\vec{r}(t) = (\cos t + t\sin t)\vec{\imath} + (\sin t - t\cos t)\vec{\jmath} + 3\vec{k}$$

97. **(C)** Find T, N, B, κ, and τ for the space curve

$$\vec{r}(t) = (\cos^3 t)\vec{\imath} + (\sin^3 t)\vec{\jmath}$$

98. (Patrick) The speedometer on your car reads a steady 35 mph. Could you be accelerating? Explain.

99. (Cherie) Can anything be said about the speed of a particle whose acceleration is always orthogonal to its velocity? Explain.

100. (Sarah) Show that a moving particle will move in a straight line if the normal component of its acceleration is zero.

101. (Ashley) Show that κ and τ are both zero for the line

$$\vec{r}(t) = (x_0 + At)\vec{\imath} + (y_0 + Bt)\vec{\jmath} + (z_0 + Ct)\vec{k}$$

102. (Clint) Show that the curvature of a smooth curve $\vec{r}(t) = f(t)\vec{\imath} + g(t)\vec{\jmath}$ defined by twice-differentiable functions $x = f(t)$ and $y = g(t)$ is given by the formula

$$\kappa = \frac{|\,\dot{x}\ddot{y} - \dot{y}\ddot{x}\,|}{(\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}}$$

103. (Amanda) Suppose that $\vec{r}(t) = f(t)\vec{\imath} + g(t)\vec{\jmath} + h(t)\vec{k}$ is twice differentiable for all t in an interval $[a, b]$, that $\vec{r} = \vec{0}$ when $t = a$ and that $\vec{v} \cdot \vec{k} = 0$ for all t in $[a, b]$. Then show that $h(t) = 0$ for all t in $[a, b]$.

104. (Amanda) Show that

$$\begin{vmatrix} x_1 - x & y_1 - y & z_1 - z \\ x_2 - x & y_2 - y & z_2 - z \\ x_3 - x & y_3 - y & z_3 - z \end{vmatrix} = 0$$

is an equation for the plane through the three noncollinear points $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, and $P_3(x_3, y_3, z_3)$.

105. (Courtney) Let P be a plane in space and let $\vec{v}$ be a vector. The vector projection of $\vec{v}$ onto the plane P, $\text{proj}_P\vec{v}$, can be defined informally as follows. Suppose that the sun is shining so that its rays are normal to the plane P. Then $\text{proj}_P\vec{v}$ is the "shadow" of $\vec{v}$ onto P. If P is the plane $x + 2y + 6z = 6$ and $\vec{v} = \vec{\imath} + \vec{\jmath} + \vec{k}$ find $\text{proj}_P\vec{v}$.

Chapter 11

106. **(C)** Be able to do problems 1-44 in Section 11.1

107. (Ashley) Does the function $f(x, y, z) = xyz$ have a maximum value on the line $x = 20 - t$, $y = t$, $z = 20$? If so, what is it? Give reasons for your answer.

108. (Sarah) Does the function $f(x, y, z) = xy - z$ have a minimum value on the line $x = t - 1$, $y = t - 2$, $z = t + 7$? If so, what is it? Give reasons for your answer.

109. **(C)** Be able to do problems 1-34 in section 11.2.

110. (Eunice) What is the $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4}{x^4 + y^2}$? If the limit does not exist, explain why.

111. (Riley / Ashley) Find $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3 - xy^2}{x^2 + y^2}$ or show that the limit dne.

112. (Sarah / Riley) Find $\lim\limits_{(x,y)\to(0,0)} \cos\left(\dfrac{x^3-y^3}{x^2+y^2}\right)$ or show that the limit dne.

113. (Patrick) If $f(x_0, y_0) = 3$ what can you say about

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y)$$

if f is continuous at (x_0, y_0)? If f is not continuous at (x_0, y_0)? Give reasons for your answer.

114. (Sarah) Does knowing that $2\mid xy\mid - \dfrac{x^2y^2}{6} < 4 - 4\cos\sqrt{\mid xy\mid} < 2\mid xy\mid$ tell you anything about

$$\lim_{(x,y)\to(0,0)} \frac{4-4\cos\sqrt{\mid xy\mid}}{\mid xy\mid}?$$

Give reasons for your answer.

115. **(C)** Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the following functions:

    (a) $f(x,y) = x^2 - xy + y^2$
    (b) $f(x,y) = (x^2-1)(y+2)$
    (c) $f(x,y) = \left(x^3 + \dfrac{y}{2}\right)^{\frac{2}{3}}$
    (d) $f(x,y) = \dfrac{x}{x^2+y^2}$
    (e) (Amanda) $f(x,y) = \tan^{-1}\left(\dfrac{y}{x}\right)$
    (f) $f(x,y) = e^{xy}\ln y$
    (g) (Amanda) $f(x,y) = \log_y x$
    (h) (Amanda) $f(x,y) = \int_x^y g(t)dt$ where $g(t)$ is continuous $\forall t$.

116. **(C)** Compute f_x, f_y and f_z for the following functions:

    (a) $f(x,y,z) = (x^2+y^2+z^2)^{-\frac{1}{2}}$
    (b) $f(x,y,z) = yz\ln(xy)$

117. **(C)** Find the partial derivatives of $h(\rho,\phi,\theta) = \rho\sin\phi\cos\theta$.

118. **(C)** Find all second-order partial derivatives of the following functions:

    (a) $h(x,y) = xe^y + y + 1$
    (b) $s(x,y) = \tan^{-1}\left(\dfrac{y}{x}\right)$

119. (Courtney) Find the value of $\frac{\partial z}{\partial x}$ at the point $(1,1,1)$ if the equation $xy + z^3x - 2yz = 0$ defines z as a function of the two independent variables x and y and the partial derivatives exist.

119.5 (Amanda) Let $f(x,y) = 0$ be an equation defining y implicitly as a function of x. Then $\frac{dy}{dx} = -\frac{f_x}{f_y}$. Prove this.

119.75 (Eunice) What is the difference between $\frac{df}{dx}$ and $\frac{\partial f}{\partial x}$? Or between $\frac{dy}{dx}$ and $\frac{\partial y}{\partial x}$?

120. (Courtney) Does a function $f(x,y)$ with continuous first partial derivatives throughout an open region R have to be continuous on R? Give reasons for your answer.

121. (Sarah) If a function $f(x,y)$ has continuous second partial derivatives throughout an open region R must the first-order partial derivatives of f be continuous on R? Give reasons for your answer.

122. **(C)** Find $\frac{dw}{dt}$ at $t=3$ if $w=\ln(x^2+y^2+z^2)$ where $x=\cos t$, $y=\sin t$ and $z=4\sqrt{t}$.

123. **(C)** Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ at $(u,v)=(1.3,\frac{\pi}{6})$ if $z=\tan^{-1}\left(\frac{x}{y}\right)$ where $x=u\cos v$ and $y=u\sin v$.

124. **(C)** Assuming that the following equations define y as a differentiable function of x, find $\frac{dy}{dx}$ at the given point.

    (a) $x^3-2y^2+xy=0$ at $(1,1)$
    (b) $xe^y+\sin xy+y-\ln 2=0$ at $(0,\ln 2)$

125. (Cherie) Assuming that $F(x,y,z)=0$ determines z as a differentiable function of x and y, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for $z^3-xy+yz+y^3-2=0$ at the point $(1,1,1)$.

126. (Ashley) Assuming that $F(x,y,z)=0$ determines z as a differentiable function of x and y, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for $xe^y+ye^z+2\ln x-2-3\ln 2=0$ at the point $(1,\ln 2,\ln 3)$.

127. (Cherie) Find $\frac{\partial w}{\partial v}$ when $u=-1$ and $v=2$ if $w=xy+\ln z$, $x=\frac{v^2}{u}$, $y=u+v$ and $z=\cos u$.

128. (Patrick) The lengths a, b, and c of the edges of a rectangular box are changing with time. At the instant in question, $a=1$ m, $b=2$ m, $c=3$ m, $\frac{da}{dt}=\frac{db}{dt}=1$ m/sec, and $\frac{dc}{dt}=-3$ m/sec. At what rates are the box's volume V and surface area S changing at that instant? Are the box's interior diagonals increasing in length or decreasing?

129. (Riley) If $f(u,v,w)$ is differentiable and $u=x-y$, $v=y-z$ and $w=z-x$ show that

$$\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}=0$$

130. (Clint) Suppose that we substitute polar coordinates $x=r\cos\theta$ and $y=r\sin\theta$ in a differentiable function $w=f(x,y)$. Show that

$$\frac{\partial w}{\partial r}=f_x\cos\theta+f_y\sin\theta \text{ and } \frac{1}{r}\frac{\partial w}{\partial \theta}=-f_x\sin\theta+f_y\cos\theta$$

131. (Courtney / Amanda) Suppose that the partial derivatives of a function $f(x,y,z)$ at points on the helix $x=\cos t$, $y=\sin t$, and $z=t$ are

$$f_x=\cos t \qquad f_y=\sin t \qquad f_z=t^2+t-2$$

At what points on the curve, if any, can f take on extreme values?

131.5 (Katie) State the Mixed Derivative Theorem (see p. 897). Why is this a big deal? In particular, find a function for which the mixed derivative theorem does not hold.

132. **(C)** Find the gradient of the function at the given point:
    (a) $f(x,y)=\ln(x^2+y^2)$ at $(1,1)$
    (b) $g(x,y)=\dfrac{x^2}{2}-\dfrac{y^2}{2}$ at $(\sqrt{2},1)$.

133. **(C)** Find ∇f at the given point:
    (a) (Joel) $f(x,y,z)=2z^3-3(x^2+y^2)z+\tan^{-1}xz$ at $(1,1,1)$
    (b) $f(x,y,z)=(x^2+y^2+z^2)^{-\frac{1}{2}}+\ln(xyz)$ at $(-1,2,-2)$

134. **(C)** find the derivative of the function at P_0 in the direction of $\vec{A}$.
    (a) (Amanda) $f(x,y)=x-\frac{y^2}{x}+\sqrt{3}\sec^{-1}(2xy)$, $P_0=(-1,1)$, $\vec{A}=4\vec{\imath}+3\vec{\jmath}$
    (b) $g(x,y,z)=3e^x\cos yz$, $P_0=(0,0,0)$, $\vec{A}=2\vec{\imath}+\vec{\jmath}-2\vec{k}$
    (c) $h(x,y,z)=\cos xy+e^{yz}+\ln zx$, $P_0=\left(1,0,\frac{1}{2}\right)$, $\vec{A}=\vec{\imath}+2\vec{\jmath}+2\vec{k}$

135. (Eunice) Find the directions in which the function $f(x,y)=x^2y+e^{xy}\sin y$ increases and decreases most rapidly at the point $P_0=(1,0)$.

136. (Ashley) Find the directions in which the function $f(x,y,z)=xe^y+z^2$ increases and decreases most rapidly at the point $P_0=\left(1,\ln 2,\dfrac{1}{2}\right)$.

137. **(C)** Find the equation of the tangent plane to the surface $\cos\pi x-x^2y-z^2=18$ at $P_0=(0,1,2)$. Also find the equation of the normal line to the surface at that point.

138. **(C)** Find the equation of the tangent plane to the surface $x^2+y^2-2xy-x+3y-z=-4$ at $P_0=(2,-3,18)$. Also find the equation of the normal line to the surface at that point.

139. **(C)** Find an equation for the plane that is tangent to the surface $z=\ln(x^2+y^2)$ at $(1,0,0)$.

140. **(C)** Find an equation for the plane that is tangent to the surface $z=\sqrt{y-x}$ at $(1,2,1)$.

141. (Eunice) In what direction is the derivative of $f(x,y)=xy+y^2$ at $P(3,2)$ equal to zero?

142. (Leslie) Is there a direction $\vec{u}$ in which the rate of change of $f(x,y)=x^2-3xy+4y^2$ at $P(1,2)$ equals 14? Give reasons for your answer.

143. (Courtney) The derivative of $f(x,y)$ at $P_0=(1,2)$ in the direction of $\vec{\imath}+\vec{\jmath}$ is $2\sqrt{2}$, and in the direction of $-2\vec{\jmath}$ is -3. what is the derivative of f in the direction of $\vec{\imath}-2\vec{\jmath}$? Give reasons for your answer.

144. (Amanda) The derivative of $f(x,y,z)$ at a point P is greatest in the direction of $\vec{v}=\vec{\imath}+\vec{\jmath}-\vec{k}$. In this direction, the value of the derivative is $2\sqrt{3}$.
    (a) What is ∇f at P? Give reasons for your answer.
    (b) What is the derivative of f at P in the direction of $\vec{\imath}+\vec{\jmath}$?

Exam 2 covers to this point

Final Exam is cumulative, but new material begins at this point.

145. **(C)** Find the linearization of
    - (a) $f(x,y) = x^2 + y^2 + 1$ at $(1,1)$.
    - (b) $f(x,y) = e^{2y-x}$ at $(1,2)$
    - (c) $f(x,y,z) = xy + yz + xz$ at $(1,1,1)$
    - (d) $f(x,y,z) = e^x + \cos(y+z)$ at $\left(0, \frac{\pi}{2}, 0\right)$

146. (Amanda) State the definition of local maximum, local minimum, and saddle points for a surface $z = f(x,y)$. Use the function $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$ to illustrate the definitions.

147. (Katie / Sarah) State and explain the First Derivative Test for Local Extreme Values. Use the function $f(x,y) = \frac{1}{x^2+y^2-1}$ to illustrate the use of the theorem.

148. (Sarah) State and explain the Second Derivative Test for Local Extreme Values. Use the function $f(x,y) = e^{2x}\cos y$ to illustrate the use of the theorem.

149. (Leslie) Find the absolute maxima and minima of $f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0$, $y = 2$, and $y = 2x$ in the first quadrant.

150. (Riley) Find the absolute maxima and minima of $f(x,y) = (4x - x^2)\cos y$ on the rectangular plate $0 \le x \le 1$, and $0 \le y \le 1$.

151. (Courtney / Sarah) Find two numbers a and b with $a \le b$ such that $\displaystyle\int_a^b 6 - x - x^2 \ \ dx$ has its largest value.

152. (Ashley) Find the critical point of $f(x,y) = xy + 2x - \ln x^2 y$ in the open first quadrant ($x > 0$ and $y > 0$) and show that f takes on a minimum there.

153. (Clint) Find the maxima, minima, and saddle points of $f(x,y)$, if any, given that $f_x = 9x^2 - 9$ and $f_y = 2y + 4$.

154. (Clint) Show that $(0,0)$ is a critical point of $f(x,y) = x^2 + kxy + y^2$ no matter what value the constant k has.

155. (Ashley) For what values of the constant k does the second derivative test guarantee that $f(x,y) = x^2 + kxy + y^2$ will have a saddle point at $(0,0)$? For what values of k is the second derivative test inconclusive? Give reasons for your answer.

156. (Patrick) State Taylor's Formula for $f(x,y)$ at (a,b). Use this to find the cubic approximations of $f(x,y) = e^x \ln(1+y)$ near the origin.

157. **(C)** Use Taylor's formula to find the quadratic approximation of $f(x,y) = \frac{1}{1-x-y}$ near the origin.

Chapter 12

158. (Eunice) Define, using sums, a double integral over a rectangle. Explain the interpretation of the double integral. In particular, remember that for a standard integral (as in Calc II) the integral is the sum of the area of the rectangles, and gives the area under the curve. Explain the similar ideas for multiple integrals.

159. **(C)** Evaluate $\int_0^3 \int_{-2}^0 (x^2y - 2xy)dy\ dx$

160. (Clint / Courtney) Find the integral of $f(x, y) = \frac{x}{y}$ over the region in the first quadrant bounded by the lines $y = x$, $y = 2x$, $x = 1$ and $x = 2$.

161. (Ashley) Find the integral of $f(s, t) = e^s \ln t$ over the region in the first quadrant of the $st-$plane that lies above the curve $s = \ln t$ from $t = 1$ to $t = 2$.

162. (Eunice) State Fubini's Theorem (first form). Use the integral

$$\int_0^{\frac{3}{2}} \int_0^{9-4x^2} 16x\ dy\ dx$$

to show that Fubini's Theorem holds.

163. (Joel) Find the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and below by the triangle enclosed by the lines $y = x$, $x = 0$ and $x + y = 2$.

164. (Riley) Find $\displaystyle\int_1^{\infty} \int_{e^{-x}}^1 \frac{1}{x^3y}\ dy\ dx$

165. (Amdanda) State the formula for area in polar coordinates. Explain where the "$r\ dr\ d\theta$" comes from.

166. **(C)** Calculate the following integrals (think polar)

    (a) $\displaystyle\int_{-1}^1 \int_0^{\sqrt{1-x^2}} dy\ dx$

    (b) $\displaystyle\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2 + y^2)\ dx\ dy$

    (c) $\displaystyle\int_0^{\ln 2} \int_0^{\sqrt{(\ln 2)^2 - y^2}} e^{\sqrt{x^2+y^2}}\ dx\ dy$

167. (Leslie) Find the area of the region common to the interiors of the cardioids $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$.

168. (Ashley) Integrate $f(x, y) = \dfrac{\ln(x^2 + y^2)}{\sqrt{x^2 + y^2}}$ over the region $1 \le x^2 + y^2 \le e$.

169. (Patrick) Write six different iterated triple integrals for the volume of the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 1$, $y = 2$ and $z = 3$. Evaluate one of the integrals.

170. **(C)** Evaluate the following:

   (a) $\displaystyle\int_0^1\int_0^1\int_0^1 (x^2+y^2+z^2)dz\ dy\ dx$

   (b) $\displaystyle\int_0^{\sqrt{2}}\int_0^{3y}\int_{x^2+3y^2}^{8-x^2-y^2} dz\ dx\ dy$

   (c) $\displaystyle\int_0^{\frac{\pi}{4}}\int_0^{\ln\sec v}\int_{-\infty}^{2t} e^x\ dx\ dt\ dv$

171. (Leslie) Find the volume of the region common to the interiors of the cylinders $x^2+y^2=1$ and $x^2+z^2=1$ (for a picture see #29 on page 1016).

172. (Sarah) Find the volume of the finite region bounded by the planes $z=x$, $x+z=8$, $z=y$, $y=8$ and $z=0$.

173. (Courtney) Find the average value of $F(x,y,z)=x+y-z$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes $x=1$, $y=1$, and $z=2$.

174. (Ashley) For what value of c is the volume of the ellipssoid $x^2+\left(\frac{y}{2}\right)^2+\left(\frac{z}{c}\right)^2=1$ equal to 8π?

175. (Amanda) State the three coordinates used in a spherical coordinate system. Provide the conversion factors from both cylindrical coordinates and rectangular coordinates into spherical coordinates. Convert the "volume elements" also.

176. **(C)** Evaluate the cylindrical coordinate integrals:

   (a) $\displaystyle\int_0^{2\pi}\int_0^1\int_r^{\sqrt{2-r^2}} dz\ r\ dr\ d\theta$

   (b) $\displaystyle\int_0^{2\pi}\int_0^{\frac{\theta}{2\pi}}\int_0^{3+24r^2} dz\ r\ dr\ d\theta$

177. (Patrick) Convert the integral

$$\int_{-1}^{1}\int_0^{\sqrt{1-y^2}}\int_0^x (x^2+y^2)\ dz\ dx\ dy$$

   into an equivalent integral in cylindrical coordinates and evaluate the result.

178. (Leslie) Find the volume of the solid bounded below by the hemisphere $\rho=1$, $z\geq 0$ and above by the cardiod of revolution $\rho=1+\cos\phi$ (See page 1034, #34 for a picture)

179. (Clint) Let D be the region in the first octant that is bounded below by the cone $\phi=\frac{\pi}{4}$ and above by the sphere $\rho=3$. Express the volume of D as a iterated triple integral in both cylindrical and spherical coordinates. Then find the volume.

180. **(C)** Find the volume of the region bounded above by the sphere $x^2=y^2+z^2=2$ and below by the paraboloid $z=x^2+y^2$.

181. **(C)** Find the volume of the solid cut from the thick-walled cylinder $1\leq x^2+y^2\leq 2$ by the cones $z=\pm\sqrt{x^2+y^2}$.

182. (Ashley) Solve the system

$$u = x - y \qquad\qquad v = 2x + y$$

for x and y in terms of u and v. Then find the value of the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$.

183. (Courtney) Use the transformation in the above problem to evaluate the integral

$$\int\int_R (2x^2 - xy - y^2)\ dx\ dy$$

for the region R in the first quadrant bounded by the lines $y = -2x + 4$, $y = -2x + 7$, $y = x - 2$, and $y = x + 1$.

184. (Patrick) Let R be the region in the first quadrant of the $xy-$plane bounded by the hyperbolas $xy = 1$, $xy = 9$, and the lines $y = x$, $y = 4x$. Use the transformation $x = \frac{u}{v}$ $y = uv$ with $u > 0$ to rewrite

$$\int\int_R \left(\sqrt{\frac{y}{x}} + \sqrt{xy}\right)\ dx\ dy$$

as an integral over an appropriate region G in the $uv-$plane. Then evaluate the $uv-$integral over G.

185. Find the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

(Hint: Let $x = au$, $y = bv$ and $z = cw$. Then find the volume of an appropriate region in $uvw-$space.)

Chapter 13

186. (Amanda) Define the line integral using sums. Explain what the formula means. Give the formula we use to calculate line integrals. Explain the process by integrating $f(x,y,z) = x - y + z - 2$ over the straight line segment from $(0,1,1)$ to $(1,0,1)$.

187. **(C)** Be able to do problems 1-6 in Section 12.1

188. **(C)** Evaluate the following line integrals over the given space curves:

    (a) $\int_C (x+y)ds$ where C is the straight line segment $x = t$, $y = 1-t$ and $z = 0$ from $(0,1,0)$ to $(1,0,0)$.

    (b) Find the line integral of $f(x,y,z) = x+y+z$ over the straight line segment from $(1,2,3)$ to $(0,0,0)$.

189. (Katie) Find the line integral of $f(x,y,z) = \dfrac{\sqrt{3}}{x^2+y^2+z^2}$ over the curve $\vec{r}(t) = t\vec{\imath} + t\vec{\jmath} + t\vec{k}$ with $1 \le t < \infty$.

190. (Cherie) Find the integral of $f(x,y) = \dfrac{x^3}{y}$ over the curve $C : y = \dfrac{x^2}{2}$ for $0 \le x \le 2$.

191. (Katie / Riley) Define the gradient vector field. Use this definition to calculate the gradient vector field of the function $g(x, y, z) = xy + yz + xz$.

192. Give a formula $\vec{F} = M(x, y)\vec{\imath} + N(x, y)\vec{\jmath}$ for the vector field in the plane that has the property that $\vec{F} = \vec{0}$ at $(0, 0)$ and that at any other point (a, b), $\vec{F}$ is tangent to the circle $x^2+y^2 = a^2+b^2$ and points in the clockwise direction with magnitude $\mid \vec{F} \mid = \sqrt{a^2 + b^2}$.

193. (Patrick) Give the six formulas for finding the work done by a force $\vec{F}$ over a smooth curve $\vec{r}(t)$. Use one of these to find the work done by the force $\vec{F} = (y + z)\vec{\imath} + (z + x)\vec{\jmath} + (x + y)\vec{k}$ over the curved path $C : t\vec{\imath} + t^2\vec{\jmath} + t^4\vec{k}$ where $0 \leq t \leq 1$.

194. (Riley) Find the work done by the force $\vec{F} = xy\vec{\imath} + y\vec{\jmath} - yz\vec{k}$ over the curve $\vec{r}(t) = t\vec{\imath} + t^2\vec{\jmath} + t\vec{k}$, $0 \leq t \leq 1$ in the direction of increasing t.

195. (Riley) Evaluate $\displaystyle\int_C xy\ dx + (x + y)\ dy$ along the curve $y = x^2$ from $(-1, 1)$ to $(2, 4)$.

196. Find the circulation and flux of the field $\vec{F} = x\vec{\imath} + y\vec{\jmath}$ around and across the closed semicircular path that consists of the semicircular arch $\vec{r_1}(t) = (a\cos t)\vec{\imath} + (a\sin t)\vec{\jmath}$, $0 \leq t \leq \pi$, followed by the line segment $\vec{r_2} = t\vec{\imath}$, $-a \leq t \leq a$.

197. (Clint) If $\vec{F} = -4xy\vec{\imath} + 8y\vec{\jmath} + 2\vec{k}$ is the velocity field of a fluid flowing through a region in space, find the flow along the curve $\vec{r}(t) = t\vec{\imath} + t^2\vec{\jmath} + \vec{k}$ in the direction of increasing t with $0 \leq t \leq 2$.

198. (Eunice) State the definitions for Path Independence and for Conservative Fields. State the Fundamental Theorem of Line Integrals.

199. (Eunice) State the Closed-Loop Property of Conservative Fields.

200. State the component test for conservative fields. Use this theorem to determine if the field $\vec{F} = (z + y)\vec{\imath} + z\vec{\jmath} + (y + x)\vec{k}$ is conservative.

201.