The Moore Method

Ron Taylor, Berry College, Fall 2003
edited by Jacqueline Jensen, Fall 2004

August 19, 2004

"The role of the Moore Method is to raise the standard of what students mean by understanding." – Mike Starbird

1 Moore

The Moore Method (or Texas Method) of mathematics instruction is a teaching/learning style propagated by Robert Lee [R.L.] Moore of The University of Texas at Austin in the first half of the 20^{th} century. Moore is considered by many to be one of the greatest American mathematics teachers and was prolific in producing students who went on to earn the Ph.D. degree in mathematics and other fields. The method can be traced back to E.H. Moore, who was R.L. Moore's advisor at the University of Chicago, but R.L. Moore was the one who made it famous. It is also related to the Socratic Method employed in American law schoools, but without the confrontation intrinsic to that method.

2 Moore's Method

In a *traditional* Moore method class the students are given a set of notes on the first day of class and told to come back the next day prepared to present their solutions to some of the problems. In the interim they are to discuss the problems with no person and to look for the solutions in no book. Moreover, students presenting problems to the class are not given class time to fix their mistakes, beyond one minute of thinking time. If they are unable to answer some question in that one minute, then they are asked to try another problem or sit down and try the same problem at a later time. Additionally, other students are not allowed to make helpful suggestions to the presenter. The reason for this is that Moore wanted his students to work independently on the problems and create the mathematics for themselves.

At the heart of this method is the idea that it is important to try, fail and try again. Our society has engendered the belief that making mistakes defines failure and weakness, and that being wrong and failing are bad. But any great innovation or new idea, whether it's a medical breakthrough, a sociological discovery or a scientific realization, occurs through a succession of failed attempts. It is through the succession of failures that things are worked out and insights are gained. Every failed attempt is a new discovery. That is part of the learning. It is this rigor that makes the Texas

method so successful in the classroom and the reason that so many of Moore's students went on to get Ph.D.'s.

3 Moore's Method – Modified (Jensen)

In this course we will be using a modified Moore method. The course will retain the emphasis on presentations and the caveat that you will discover the mathematics yourself. You will have a textbook for your consideration, and it will be useful for definitions, but you should discover and develop solutions to the problems, and presenting the book's solutions as your own is *completely unacceptable*. You will learn and understand the mathematics better if you solve the problems on your own rather than parroting someone else's solutions. However, you will *NOT* be allowed to discuss the problems with other members of the class. If you find yourself stuck that's why they invented office hours. But, any problem you present to the class should be your own work and written in your own words. When you present a problem, you will be asserting that you have neither given nor recieved any unauthorized help on the work and that you understand the argument you are presenting.

Because this course will also have exams, we will make a distinction between preparing problems for presentations and preparing for exams. While studying for exams, you are welcome and encouraged to form study groups. The best way to decide whether or not you are able to discuss the problems is the following: if a problem has been discussed in class, or is clearly labeled a computational problem, you may discuss it with classmates. In particular, exams will cover material presented in class by the professor and other students, and you are allowed to discuss this material with classmates and others.

During class presentations keep in mind that you are not presenting a proof for the instructor of the course. You are presenting proof that should convince your classmates, and yourself, of the truth (or falsehood) of a particular claim. Of course, you should be able to convice the instructor as well, but that is not your primary audience. Inasmuch as you are relative novices to the proofwriting process, you should expect to write in a way that convinces other non-experts without relying on slang, jargon or colloquial phrases. One of the goals of this course is to give you the opportunity to learn to use mathematical language correctly and precisely. To that end, when it is your turn to present something at the board you shoud bring a complete, carefully organized version of your solution with you for reference. (You will catch a lot of your own mistakes while writing this out.) Whenever someone asks you a question while you are presenting you should view this as an opportunity to help that person better understand the proof. If someone points out a mistake, you should view this as an opportunity to make your argument better or in some cases be correct.

On the other hand, when you are watching someone present a proof to the class and you do not understand you should ask a question. Chances are that if you have a question, then someone else in the class has a similar question. If you didn't need to ask another person's question, then you should be able to answer it. But don't always assume that you understand what someone is saying just because it seems to make sense when you first hear it. You should always be asking yourself *What does this really mean?* and *How do I know it's true?* In the spirit of Understand First – Critique Second, comments about the proof that might make it more slick should be saved for after the presentation.

4 More on the Modified Moore Method

Any class using a Moore-like method will be driven primarily by student presentations. Hence, during class time, the role of the teacher is that of knowledgeable moderator instead of a traditional lecturer. Ordinarily the teacher is there to explain definitions, answer general questions and act as a referee. The teacher will also observe presentations and student reactions to the presentations and occasionally ask questions either to point out a flaw in your argument or to check if you are really confident that your solution is correct. That is, if the teacher asks if you're sure about something you may be wrong or right and it is up to you to decide. But, for the most part, it is the job of the students to critique the presentations. This is not meant to be a forum for criticizing another person's presentation skills or their level of mathematical maturity. Rather it is an opportunity for you to help your classmates be better mathematicians and mathematics students. The dynamics of class interaction are thus very important. Since the class will consist largely of your contributions, the way you conduct yourself in class will determine whether the course is a success or failure. The goal is to foster a spirit of friendly competition and neighborly rivalry. You should strive to push your classmates to succeed.

When you present a problem to the class, you should write a clear and well-organized account on the board. This written evidence of your thinking is the way that a verdict on your presentation can be reached. Begin by writing at least the number of the problem and (if appropriate) whether you intend to prove it true or false. Write down the steps of your argument clearly and in the proper order. Abbreviations are acceptable if standard or clearly explained, but it is important to write connecting words (if...then, implies, and, or, whenever,...) between equations so that the class can follow the logic of your argument. "Using good language and good notation can greatly facilitate the learning of mathematics..." [8] Also, you should be prepared to explain what you have done, and to answer questions.

If you have done a problem several days before you present it, it is a good idea to review it just before the class in which you think it will come up. Otherwise you may find yourself standing at the board staring blankly at a sheet of paper covered with meaningless scribbles.

When you watch someone else's presentation you should be skeptical, but willing to be convinced. It is imperative that you understand what goes on in class, so it is even more important than usual to ask questions. We are a community of learners, and no stigma can ever be attached to any genuine question, no matter how "stupid" it may seem to the person asking it.

It is also part of your duty to the community to ask questions when something is unclear, or to point out what seems to you to be a gap or mistake in the argument. Of course it goes without saying (almost) that you should behave as you would wish others to behave when you are at the board. (That does not mean for you to remain silent.) It is not good form to ask artificial questions, to try to trip people up, or otherwise show off. Remember that it is arguments, and not people, that can be either correct or incorrect. Hence, you should try to learn to analyze arguments objectively, and leave your ego out of it. Don't be too quick to think you are wrong, and don't be unable to see when you have made a mistake. There is such a thing as objective truth, independent of who we are. Try to find out what it is. One way to do this is to learn to question your own proofs, and then to answer those questions. Two important objectives of this course is for you to develop your creative and critical thinking skills and to gain confidence in your own abilities.

5 Making the Most of the Modified Moore Method

Your enrollment in this course makes certain statements about you. You enjoy mathematics. You are taking this course because you want to, if only to satisfy a requirement. You want to learn the material of this course, at least to the point of earning a passsing grade. You are willing to work hard and study. You are prepared to do, throughout the semester, at least two hours of diligent work outside class per hour spent in class. This last point bears emphasizing. You have heard it over and over throughout your college career and you are smart enough to have been able to get away with much less in some of your previous classes at Sam Houston State. However, with this class it really is a minimum standard.

Finally, a word about the homework. The homework is probably the most important part of the course and therefore, you should strive to do as many of the problems as you can. The notes are designed to give you the best idea of how the subject matter fits together and a complete understanding of the course will be difficult if you have not given each problem due consideration.

> "It is the teacher's job to help the student [understand], and so – as you would not conceal a health problem from a doctor – you should not conceal from the teacher any faults in your understanding."[2]

References

[1] Ed Burger, transcript of interview with The Futures Channel.

[2] Nets Katz, course website, `www.math.wustl.edu/∼nets/math452.html`.

[3] Robert M. Kauffman, *Advanced Calculus*, The Educational Advancement Foundation, Austin, TX, 2003.

[4] Ted Mahavier, *A Gentler Discovery Method*, preprint.

[5] Lee May, unpublished Moore method notes.

[6] R. L. Moore, *Challenge in the Classroom*, educational video, Mathematical Association of America, Wachington, D.C., 1966.

[7] Ed Parker, unpublished Moore method notes.

[8] Tom Read, unpublished Moore method notes.

[9] Cornelius Stallmann, quoted in *A Moore Method Calculus II Course* by William S. Mahavier, The Educational Advancement Foundation, Austin, TX, 2003.