

Math 244  
Fall, 2003  
Exam 1  
Chapters 8 and §§9.1-9.3

Name: \_\_\_\_\_

Please write your name only on this page of the exam.

Show all work to receive credit.

Please leave all answers in exact value  
(i.e. no decimal approximations).

<b>Page</b>	<b>Points Possible</b>	<b>Points Earned</b>
2	12	
3	8	
4	11	
5	10	
6	17	
7	10	
8	14	
9	18	
<b>Total</b>	<b>100</b>	

The following were problems about series

1. (5 points)
2. (7 points)
3. (8 points)
4. (6 points)
5. (5 points)
6. (10 points)
7. For the following problems, let  $\vec{v} = 2\vec{i} + 10\vec{j}$  and  $\vec{u} = 2\vec{i} + 2\vec{j}$ .
  - (a) (4 points) Find  $2\vec{u} - 3\vec{v}$  in component form.
  - (b) (4 points) Find  $|\vec{u}|$  and  $|\vec{v}|$ .
  - (c) (4 points) Find  $\vec{u} \cdot \vec{v}$
  - (d) (5 points) Find the cosine of the angle between  $\vec{u}$  and  $\vec{v}$ .
  - (e) (5 points) Find  $\text{proj}_{\vec{v}}\vec{u}$
  - (f) (5 points) Write  $\vec{u}$  as the sum of a vector parallel to  $\vec{v}$  and a vector perpendicular to  $\vec{v}$ .
8. Let the position vector of a bug be given by  $\vec{r}(t) = (\sin t)\vec{i} + t\vec{j}$  for  $t \geq 0$ .
  - (a) (4 points) Find the velocity and acceleration vectors.
  - (b) (4 points) Find the speed of the bug at time  $t = \frac{\pi}{2}$ .
  - (c) (6 points) For what values of  $t$  are the velocity vector and the acceleration vector orthogonal?
9. (8 points) Calculate  $\int_0^1 ((3 + 6t)\vec{i} + (6\pi \cos \pi t)\vec{j}) dt$
10. (10 points) Solve the initial value problem

$$\frac{d\vec{r}}{dt} = \frac{1}{t}\vec{i} + \frac{1}{5-t}\vec{j} \text{ where } \vec{r}(1) = 2\vec{i} - (\ln 4)\vec{j}$$

Math 244  
Fall, 2003  
Exam 2  
Chapters 9 and §§10.1 - 10.5

Name: \_\_\_\_\_

Please write your name only on this page of the exam.

Show all work to receive credit.

Please leave all answers in exact value  
(i.e. no decimal approximations).

Page	Points Possible	Points Earned
2	24	
3	18	
4	20	
5	20	
6	18	
Total	100	

**True / False. (2 points each)** For the following problems, circle T if the statement is always true and F if the statement is sometimes false. No partial credit will be awarded for this portion of the exam.

1. T      F      Two non-zero vectors are parallel if and only if  $\vec{v} \cdot \vec{u} = 0$ .
2. T      F       $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
3. T      F       $(\vec{u} \times \vec{u}) \cdot \vec{u} = 0$
4. T      F       $(\vec{u} \times \vec{v}) \cdot \vec{v} = 0$
5. T      F       $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$
6. T      F       $(\vec{u} \times \vec{v}) \times \vec{w}$  makes sense.
7. T      F       $\vec{u} \times (-\vec{u}) = \vec{0}$
8. T      F       $\vec{u} \cdot (-\vec{u}) = 0$

**For the remainder of the exam, show all work to receive credit.**

9. For the following problems, let

$$\begin{aligned}\vec{u} &= \vec{i} + \vec{j} - 2\vec{k} \\ \vec{v} &= \vec{i} - \vec{k} \\ \vec{w} &= 2\vec{i} + 4\vec{j} - 2\vec{k}\end{aligned}$$

- (a) Find  $|\vec{u}|$
  - (b) Find  $\vec{u} \cdot \vec{w}$
  - (c) (6 points) Find  $\vec{u} \times \vec{w}$
  - (d) (3 points) Find  $\vec{v} \times \vec{u}$
  - (e) (8 points) Find the area of the triangle with sides  $\vec{u}$  and  $\vec{w}$  and  $\vec{u} + \vec{w}$
  - (f) (8 points) Find the volume of the parallelepiped determined by  $\vec{u}$ ,  $\vec{v}$  and  $\vec{w}$ .
  - (g) (8 points) Find the vector projection of  $\vec{v}$  onto  $\vec{w}$
10. (10 points) Find  $b$  so that the vectors  $\langle -b, b, 2 \rangle$  and  $\langle 1, b, -3 \rangle$  are orthogonal.
  11. (5 points) Convert the polar equation  $r = 4 \tan \theta \sec \theta$  into a Cartesian equation.
  12. (10 points) Find the length of the polar curve  $r = 2 \sin \theta + 2 \cos \theta$  from  $0 \leq \theta \leq \frac{\pi}{2}$
  13. (10 points) Prove that if  $\vec{r}$  is a differentiable vector function of  $t$  of constant length then

$$\vec{r}(t) \cdot \frac{d\vec{r}}{dt} = 0$$

14. (10 points) Solve the differential equation

$$\frac{d^2\vec{r}}{dt^2} = -32\vec{k}$$

with initial conditions

$$\vec{r}(0) = 100\vec{k} \quad \text{and} \quad \left. \frac{d\vec{r}}{dt} \right|_{t=0} = 8\vec{i} + 8\vec{j}$$

15. (10 points) Find the vector equation of the tangent line to the curve

$$\vec{r}(t) = (\sin t)\vec{i} + (t^2 - \cos t)\vec{j} + e^t\vec{k} \quad \text{at } t_0 = 0.$$

16. (12 points) A baseball is thrown from the stands 32 feet above the field at an angle of  $30^\circ$  up from the horizontal. When and how far away will the ball strike the ground if its initial speed is 32 ft/sec?