

1. Calculate the derivatives of the following functions. Wherever necessary, assume that y is a function of x .

(a) $y = \sqrt{x^2 + e^x}$

$$y' = \frac{1}{2} (x^2 + e^x)^{-\frac{1}{2}} (2x + e^x)$$

(b) $y = \ln x^{500}$

$$y = 500 \ln x$$
$$y' = 500 \cdot \frac{1}{x} = \frac{500}{x}$$

(c) $\cos(xy) = \sin(xy)$

$$-\sin(xy) \left(y + x \frac{dy}{dx} \right) = \cos(xy) \left(y + x \frac{dy}{dx} \right)$$
$$-y \sin(xy) - x \sin(xy) \frac{dy}{dx} = y \cos(xy) + x \cos(xy) \frac{dy}{dx}$$
$$-y \sin(xy) - y \cos(xy) = x \cos(xy) \frac{dy}{dx} + x \sin(xy) \frac{dy}{dx}$$
$$-y \sin(xy) - y \cos(xy) = (x \cos(xy) + x \sin(xy)) \frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{-y \sin(xy) - y \cos(xy)}{x \cos(xy) + x \sin(xy)}$$

(d) $\frac{y^2 + 5 \tan x - \frac{x}{y}}{x^4} = x$

$$y^2 + 5 \tan x - \frac{x}{y} = x^5$$
$$2y \frac{dy}{dx} + 5 \sec^2 x - \frac{y - x \frac{dy}{dx}}{y^2} = 5x^4$$
$$2y \frac{dy}{dx} + 5 \sec^2 x - \frac{1}{y} + \frac{x}{y^2} \frac{dy}{dx} = 5x^4$$
$$\left(2y + \frac{x}{y^2} \right) \frac{dy}{dx} = 5x^4 - 5 \sec^2 x + \frac{1}{y}$$
$$\frac{dy}{dx} = \frac{5x^4 - 5 \sec^2 x + \frac{1}{y}}{2y + \frac{x}{y^2}}$$

(e) $24x^3 - 3x^{-7} = y$

$$y' = 72x^2 + 21x^{-8}$$

(f) $f(x) = (e^x + \arcsin x)\left(\frac{3}{x^2}\right)$

$$f'(x) = \left(e^x + \frac{1}{\sqrt{1-x^2}}\right)\left(\frac{3}{x^2}\right) + (e^x + \sin^{-1} x)\left(\frac{-6}{x^3}\right)$$

(g) $y = x^3 - 3x^2 - 1$

$$y' = 3x^2 - 6x$$

(h) $f(x) = 7x - 3$

$$f'(x) = 7$$

(i) $f(x) = e^\pi$

$$f'(x) = 0$$

(j) $f(x) = \frac{1}{\sqrt{x}}$

$$f'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$$

(k) $f(x) = 5\sqrt[3]{x} + 6x^2 - e$

$$f'(x) = \frac{5}{3}x^{-\frac{2}{3}} + 12x$$

(l) $f(x) = \frac{1}{x^2} + \frac{1}{x} + 1$

$$f'(x) = -\frac{2}{x^3} - \frac{1}{x^2}$$

(m) $f(x) = \sin 3$

$$f'(x) = 0$$

(n) $f(x) = x^2 \sin 2$

$$f'(x) = (2 \sin 2)x$$

(o) $f(x) = x^e + x^\pi$

$$f'(x) = ex^{e-1} + \pi x^{\pi-1}$$

2. Compute the following derivatives:

(a) $\frac{d}{dx}\left(\frac{x^3}{2}\right)$

$$= \frac{3}{2}x^2$$

$$(b) \frac{d}{dx} \left(\frac{3x+2}{2x-11} \right) \\ = \frac{3(2x-11) - 2(3x+2)}{(2x-11)^2} = \frac{6x-33-6x-4}{(2x-11)^2} = \frac{-37}{(2x-11)^2}$$

$$(c) \frac{d}{dx} (\sqrt{\sin^2 x}) \\ = \frac{d}{dx} \sin x = \cos x$$

$$(d) \frac{d}{dx} ((\sin^2(2x) + 1)^e) \\ = e(\sin^2(2x) + 1)^{e-1} (2 \sin(2x) \cdot 2) \\ = 4e \sin(2x) (\sin^2(2x) + 1)^{e-1}$$

$$(e) \frac{d}{dx} (\tan(\cos(\sin x))) \\ = \sec^2(\cos(\sin x)) (-\sin(\sin x)) (\cos x)$$

$$(f) \frac{d}{dx} ((\tan x)(\cos x)(\sin x)) \\ = \sec^2 x \cos x \sin x - \tan x \sin^2 x + \tan x \cos^2 x$$

$$(g) \frac{d}{dx} ((4x+3)^4(x+1)^{-1}) \\ = 4(4x+3)^3 \cdot 4 \cdot (x+1)^{-1} - (x+1)^{-2}(4x+3)^4$$

$$(h) \frac{d}{dx} \left(\sin \left(\frac{3\pi x}{x} \right) + \cos \left(\frac{3\pi x}{2} \right) \right) \\ = -\sin \left(\frac{3\pi x}{2} \right) \cdot \frac{3\pi}{2}$$

$$(i) \frac{d}{dx} (x \tan x) \\ = \tan x + x \sec^2 x$$

$$(j) \frac{d}{dx} ((x^2 + 1) \sec x) \\ = 2x \sec x + (x^2 + 1) \sec x \tan x$$

$$(k) \frac{d}{dx} (x(x^2 + 1) \tan x \sec x) \\ = \frac{d}{dx} ((x^3 + x) \tan x \sec x) \\ = (3x^2 + 1) \tan x \sec x + (x^3 + x) \sec^3 x + (x^3 + x) \sec x \tan^2 x$$

$$(l) \frac{d}{dx} \left(x \sin^{-1} x + \sqrt{-x^2} \right) \\ = \sin^{-1} x + \frac{x}{\sqrt{1-x^2}} + \frac{1}{2} (-x^2)^{-\frac{1}{2}} (-2x) \\ = \sin^{-1} x + \frac{x}{\sqrt{1-x^2}} + -\frac{x}{\sqrt{-x^2}}$$

$$(m) \frac{d}{dt} (\cot^{-1} \sqrt{t-1})$$

$$\begin{aligned} &= -\frac{1}{1+\sqrt{t-1}^2} \left(\frac{1}{2}(t-1)^{-\frac{1}{2}} \right) = \frac{1}{t} \frac{1}{2} \frac{1}{\sqrt{t-1}} \\ &= \frac{1}{2t\sqrt{t-1}} \end{aligned}$$

$$(n) \frac{d}{ds} (\tan^{-1} \sqrt{x^2-1} + \sin^{-1} x)$$

$$\begin{aligned} &= \frac{1}{1+\sqrt{x^2-1}^2} \frac{1}{2}(x^2-1)^{-\frac{1}{2}} + \frac{1}{\sqrt{1-x^2}} \\ &= \frac{1}{1+x^2-1} \frac{1}{2\sqrt{x^2-1}} + \frac{1}{\sqrt{1-x^2}} \\ &= \frac{1}{2x^2\sqrt{x^2-1}} + \frac{1}{\sqrt{1-x^2}} \end{aligned}$$

$$(o) \frac{d}{dx} (3^{\sin x})$$

$$= 3^{\sin x} \ln 3 \cos x$$

$$(p) \frac{d}{dx} (\ln(x^2+1))$$

$$= \frac{2x}{x^2+1}$$

$$(q) \frac{d}{dx} (e^{\sin x})$$

$$= e^{\sin x} \cos x$$

$$(r) \frac{d}{dx} (x^{\sin x})$$

$$y = x^{\sin x}$$

$$\ln y = \sin x \ln x$$

$$\frac{1}{y} y' = \cos x \ln x + \frac{\sin x}{x}$$

$$y' = x^{\sin x} \left(\cos x \ln x + \frac{\sin x}{x} \right)$$

$$(s) \frac{d}{dx} \left(\sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}} \right)$$

$$y = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}$$

$$\ln y = \ln \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}$$

$$\ln y = \frac{1}{5} \ln \frac{(x-3)^4(x^2+1)}{(2x+5)^3}$$

$$\ln y = \frac{1}{5} (\ln(x-3)^4(x^2+1) - \ln(2x+5)^3)$$

$$\ln y = \frac{1}{5} (\ln(x-3)^4 + \ln(x^2+1) - 3\ln(2x+5))$$

$$\ln y = \frac{4}{5} \ln(x-3) + \frac{1}{5} \ln(x^2+1) - \frac{3}{5} \ln(2x+5)$$

$$\frac{1}{y} y' = \frac{4}{5} \frac{1}{x-3} + \frac{1}{5} \frac{2x}{x^2+1} - \frac{3}{5} \frac{2}{2x+5}$$

$$y' = y \left(\frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x+5)} \right)$$

$$y' = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}} \left(\frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x+5)} \right)$$

(t) $\frac{d}{dx} (x \sin x + e^x + \pi^x + x^\pi)$

$$= \sin x + x \cos x + e^x + \pi^x \ln \pi + \pi x^{\pi-1}$$

(u) $\frac{d}{dx} (\ln 2 \cdot \log_2 x)$

$$= \frac{d}{dx} \left(\ln 2 \cdot \frac{\ln x}{\ln 2} \right)$$

$$= \frac{d}{dx} (\ln x)$$

$$= \frac{1}{x}$$

(v) $\frac{d}{dx} (\ln 10^x)$

$$= \frac{d}{dx} (x \ln 10)$$

$$= \ln 10$$

(w) $\frac{d}{dt} \left(\frac{t^2 + 3t}{t} \right)$

$$= \frac{(2t+3)t - (t^2+3t)}{t^2}$$

$$= \frac{2t^2 + 3t - t^2 - 3t}{t^2}$$

$$= \frac{t^2}{t^2}$$

$$= 1$$

(x) $\frac{d}{dx} (x \ln x)$

$$= \ln x + 1$$

$$\begin{aligned}
\text{(y)} \quad \frac{d}{dx} \left(\frac{e^x}{e^{-x} + 1} \right) &= \frac{e^x(e^{-x} + 1) - (-e^{-x})(e^x)}{(e^{-x} + 1)^2} \\
&= \frac{1 + e^x + 1}{(e^{-x} + 1)^2} \\
&= \frac{2 + e^x}{(e^{-x} + 1)^2}
\end{aligned}$$

3. Find $\frac{dy}{dx}$ for each of the following;

(a) $xy + 2x + 3y = 1$

$$\begin{aligned}
xy + 3y &= 1 - 2x \\
y(x + 3) &= 1 - 2x \\
y &= \frac{1 - 2x}{x + 3} \\
\frac{dy}{dx} &= \frac{-2(x + 3) - (1 - 2x)}{(x + 3)^2} \\
&= \frac{-2x - 6 - 1 + 2x}{(x + 3)^2} \\
&= \frac{-7}{(x + 3)^2}
\end{aligned}$$

(b) $\sqrt{xy} + y^2 = 1$

$$\begin{aligned}
\frac{1}{2}(xy)^{-\frac{1}{2}} \left(y + \frac{dy}{dx}x \right) + 2y \frac{dy}{dx} &= 1 \\
\frac{1}{2\sqrt{xy}} \left(y + \frac{dy}{dx}x \right) + 2y \frac{dy}{dx} &= 1 \\
\frac{y}{2\sqrt{xy}} + \frac{x}{2\sqrt{xy}} \frac{dy}{dx} + 2y \frac{dy}{dx} &= 1 \\
\frac{x}{2\sqrt{xy}} \frac{dy}{dx} + 2y \frac{dy}{dx} &= 1 - \frac{y}{2\sqrt{xy}} \\
\frac{dy}{dx} \left(\frac{x}{2\sqrt{xy}} + 2y \right) &= 1 - \frac{y}{2\sqrt{xy}} \\
\frac{dy}{dx} &= \frac{1 - \frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}} + 2y}
\end{aligned}$$

$$(c) \sin^2 x + \cos^2 y = 1$$

$$2 \sin x \cos x - 2 \cos y \sin y \frac{dy}{dx} = 0$$

$$2 \sin x \cos x = 2 \cos y \sin y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2 \sin x \cos x}{2 \cos y \sin y}$$

$$(d) \frac{x^3 - xy}{x} = 3y$$

$$x^2 - y = 3y$$

$$2x - \frac{dy}{dx} = 3 \frac{dy}{dx}$$

$$2x = 2 \frac{dy}{dx}$$

$$\frac{dy}{dx} = x$$

$$(e) x(t) = 2t^7 - 4t^2 + 5$$

$$y(t) = 3t^2 + 5$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy}{dt} = 6t$$

$$\frac{dx}{dt} = 14t^6 - 8t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6t}{14t^6 - 8t}$$

$$(f) x(t) = \cos(5t^2 + t)$$

$$y(t) = \cos^2(7t^3 - t^2)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy}{dt} = 2 \cos(7t^3 - t^2) (-\sin(7t^3 - t^2) (21t^2 - 2t))$$

$$= -2(21t^2 - 2t) \cos(7t^3 - t^2) \sin(7t^3 - t^2)$$

$$\frac{dx}{dt} = -\sin(5t^2 + t) (10t + 1)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2(21t^2 - 2t) \cos(7t^3 - t^2) \sin(7t^3 - t^2)}{(10t + 1) \sin(5t^2 + t)}$$