Math 199
Fall, 2004

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1. You are given \$144 in one, five, and ten dollar bills. There are two more ten dollar bills than five dollar bills. How many bills of each type are there?

   We need one more piece of information: let's use "There are a total of 35 bills".

   Let x = number of one dollar bills

   y = number of five dollar bills

   z = number of ten dollar bills

   We get the equations:

$$x + y + z = 35$$
$$x + 5y + 10z = 144$$
$$z = y + 2 \Rightarrow z - y = 2$$

   We can use the third equation to rewrite the first two:

$$x + y + y + 2 = 35 \Rightarrow x + 2y = 33$$
$$x + 5y + 10(y + 2) = 144 \Rightarrow x + 5y + 10y + 20 = 144 \Rightarrow x + 15y = 124$$
$$x = y + 2$$

   Solve the first equation for x: $x = 33 - 2y$ and substitute into the second equation:

$$33 - 2y + 15y = 91$$
$$13y = 91$$
$$y = 7$$

$$x = 33 - 2y = 33 - 2(7) = 19$$

$$z = y + 2 = 7 + 2 = 9$$

   therefore there are 19 ones, 7 fives, and 9 tens.

2. In deciding whether to set up a new manufacturing plant, company analysts have decided that a linear function is a reasonable estimation for the total cost $C(x)$ in dollars to produce x items. They estimate the cost to produce 10,000 items as \$547,500 and the cost to produce 50,000 items as \$737,500.

(a) Find a formula for $C(x)$.

We know two points on the cost function:

$$C(10,000) = 547,500$$
$$C(50,000) = 737,500$$

Since the cost function is linear we can use these two points to find the marginal cost, which will be the slope of the linear function:

$$m = \frac{737,500 - 547,500}{50,000 - 10,000} = \frac{190,000}{40,000} = 4.75$$

Therefore, we have the cost function looks like:

$$C(x) = 4.75x + f$$

where f is the fixed costs. Pick a point to plug into this equation to solve for f:

$$577,500 = 4.75(10,000) + f$$
$$547,500 = 47,500 + f$$
$$f = 500,000$$

Therefore the linear cost function is

$$C(x) = 4.75x + 500,000$$

(b) Find the total cost to produce 100,000 items.

$$C(100,000) = 4.75(100,000) + 500,000 = 975,000$$

(c) Find the marginal cost of the items produced in this plant.

This is the slope of the cost function, so 4.75 per item.

3. Colleen Davis owns a factory that manufactures souvenir key chains. her weekly profit (in hundred of dollars) is given by $P(x) = -2x^2 + 60x - 120$, where x is the number of cases of key chains sold.

(a) What is the largest number of cases she can sell and still make a profit?

$$P(x) = -2x^2 + 60x - 120$$

We want $P(x) > 0$:

$$-2x^2 + 60x - 120 > 0$$
$$-2(x^2 - 30x + 60) > 0$$
$$x^2 - 30x + 60 < 0$$

Think:

$$x^2 - 30x + 60 = 0$$

$$
\begin{aligned}
x &= \frac{30 \pm \sqrt{900 - 4(1)(60)}}{2(1)} \\
&= \frac{30 \pm \sqrt{660}}{2} \\
&= 15 \pm \sqrt{165}
\end{aligned}
$$

The highest input where the expression $x^2 - 30x + 60$ is negative is

$$x = 15 + \sqrt{165} \approx 40.69$$

Therefore the largest number of cases that she can sell and still make a profit is 40

(b) How many cases should she make and sell in order to maximize her profits?
This occurs at the vertex of the function:

$$
\begin{aligned}
P(x) &= -2x^2 + 60x - 120 \\
&= -2(x^2 - 30x) - 120 \\
&= -2(x^2 - 30x + 225 - 225) - 120 \\
&= -2((x - 15)^2 - 225) - 120 \\
&= -2(x - 15)^2 + 450 - 120 \\
&= -2(x - 15)^2 + 330
\end{aligned}
$$

Therefore she should sell 15 cases to maximize her profits.

(c) What is the maximum profit she could earn?
The maximum profit she could earn is \$330.

4. The profit (in millions of dollars) from the sale of x million units of Blue Glue is given by $p = .7x - 25.5$. The cost is given by $c = .9x + 25.5$.

(a) Find the revenue equation.

$$
\begin{aligned}
p &= r - c \\
7x - 25.5 &= r - (.9x + 25.5) \\
7x - 25.5 &= r - .9x - 25.5 \\
1.6x - 25.5 + 25.5 &= r \\
r(x) &= 1.6x
\end{aligned}
$$

(b) What is the revenue from selling 10 million units?

$$r(10) = 1.6(10) = 16\text{million dollars}$$

(c) What is the break even point?
This is where profit is 0.

$$
\begin{aligned}
.7x - 25.5 &= 0 \\
.7x &= 25.5 \\
x &\approx 36.42857143
\end{aligned}
$$

or approximately $36,428,571$ units