Math 199
Fall, 2004

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1. Mc Frugal Snack Shops plan to hire two public relations firms to survey 500 customers by phone, 750 by mail, and 250 by in-person interviews. The Garcia firm has personnel to do 10 phone surveys, 30 mail surveys, and 5 interviews per hour. The Wong firm can handle 20 phone surveys, 10 mail surveys and 10 interviews per hour. For how many hours should each firm be hired to produce the exact number of surveys needed?

   Let $x =$ number of hours for the Garcia firm

   $y =$ number of hours for the Wong firm

$$10x + 20y = 500$$
$$30x + 10y = 750$$
$$5x + 10y = 250$$

   The Garcia firm should be hired for 20 hours and the Wong firm should be hired for 15 hours.

2. An animal breeder can buy four types of tiger food. Each case of Brand A contains 25 units of fiber, 30 units of protein and 30 units of fat. Each case of Brand B contains 50 units of fiber, 30 units of protein, and 20 units of fat. Each case of Brand C contains 75 units of fiber, 30 units of protein, and 20 units of fat. Each case of Brand D contains 100 units of fiber, 60 units of protein, and 30 units of fat. how many cases of each brand should the breeder mix together to obtain a food that provides 1200 units of fiber, 600 units of protein, and 400 units of fat?

   Let $a =$number of cases of Brand A

   $b =$ number of cases of Brand B

   $c =$ number of cases of Brand C

   $d =$ number of cases of Brand D

$$25a + 50b + 75c + 100d = 1200$$
$$30a + 30b + 30c + 60d = 600$$
$$30a + 20b + 20c + 30d = 400$$

   Reducing the matrix you get:

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 4 & 12 \\ 0 & 0 & 1 & -1 & 8 \end{array}\right]$$

   This is a dependent system with equations:

$$a - d = 0$$
$$b + 4d = 12$$
$$c - d = 8$$

Solving these for expressions in terms of d:

$$a = d$$
$$b = 12 - 4d$$
$$c = 8 + d$$

Therefore the solution is all points of the form:

$$(d, -4d + 12, d + 8, d)$$

but in order for the second term to be positive, $d = 0, 1, 2$ or 3. Therefore any of the following solutions work:

0 cases of A, 12 cases of B, 8 cases of C, and 0 cases of D

or

1 case of A, 8 cases of B, 9 cases of C, 1 case of D

or

2 cases of A, 4 cases of B, 10 cases of C, 2 cases of D

or

3 cases of A, 0 cases of B, 11 cases of C, 3 cases of D

3. An investment firm recommends that a client invest in AAA, A, and B rated bonds. The average yield on AAA bonds is 6%, on A bonds 7%, and on B bonds 10%. The client wants to invest twice as much in AAA bonds as in B bonds. How much should be invested in each type of bond if the total investment is $25,000, and the investor wants an annual return of $1810 on the three investments?

   Let x = amount invested in AAA bonds

   y = amount invested in A bonds

   z = amount invested in B bonds

$$x + y + z = 25,000$$
$$.06x + .07y + .1z = 1810$$
$$x = 2z$$

   Solving this system, we see that the client should invest $12,000 in AAA bonds at 6%, $7000 in A bonds at 7%, and $6000 in B bonds at 10%.

4. Chalon Bridges deposits semiannual payments of #3200 received in payment of a debt, in an ordinary annuity at 6.8% compounded semiannually. Find the final amount in the account and the interest earned at the end of 3.5 years.

$$S = 3200\left(\frac{\left(1+\frac{.068}{2}\right)^7 - 1}{\frac{.068}{2}}\right)$$
$$\approx \$24,818.76$$

The final amount in the account will be $24,818.76 and the interest earned will be

$$\$24,818.76 - 7(\$3200) = \$2418.76$$

5. In 1995, Oseola McCarthy donated $150,000 to the University of Southern Mississippi to establish a scholarship fund. What is unusual about her is that the entire amount came from what she was able to save each month from her work as a washerwoman, a job she began in 1916 at the age of 8, when she dropped out of school. How much would Ms. McCarthy have to put into her savings account at the end of every 3 months to accumulate $150,000 over 79 years? Assume that she received an interest rate of 5.25% compounded quarterly.

$$150,000 = R\left(\frac{\left(1+\frac{.075}{12}\right)^{316} - 1}{\frac{.075}{12}}\right)$$
$$R \approx 32.49$$

She would have to put $32.49 in savings account every quarter (notice 316 = 4(79))

6. The Taggert family bought a house for $91,000. They paid $20,000 down and took out a 30 year mortgage for the balance at 9%.

   (a) Find their monthly payment.

   They are going to mortgage $71,000.

$$71,000 = R\left(\frac{\left(1+\frac{.09}{12}\right)^{30\cdot 12} - 1}{\frac{.09}{12}}\right)$$
$$R = \$571.28$$

   The monthly payment for this mortgage is $571.28.

   (b) How much of the first payment was interest?

   To find the amount of the first payment that goes to interest, use $I = Prt$

$$I = (71,000)\left(\frac{.075}{12}\right)(1) = 532.50$$

   After 180 payments, the family sells their house for $136,000. They must pay closing costs of $3700 plus 2.5% of the sale price.

(c) Estimate the current mortgage balance at the time of the sale.
Since 180 payments were made, there are 180 payments remaining, so the present value is

$$\begin{aligned} P &= 571.29\left(\frac{1-\left(1+\frac{.09}{12}\right)^{-180}}{\frac{.09}{12}}\right) \\ &= 56,324,44 \end{aligned}$$

so the remaining balance is \$56,324.44.

(d) Find the total closing costs.

$$= \$3700 + (.025)(\$136,000) \approx \$7100$$

(e) Find the amount of money they receive from the sale after paying off the mortgage.
Amount of money received = selling price - closing costs - current mortgage balance

$$= 136,000 - 7100 - 56,324.44 = \$72,575.56$$

7. The *New York Times* posed a scenario with two individuals, Sue and Joe, who each have \$1200 a month to spend on housing and investing. Each takes out a mortgage for \$140,000. Sue gets a 30-year mortgage at a rate of 6.625%. Joe gets a 15-year mortgage at a rate of 6.25%. Whatever money is left after the mortgage payment is invested in a mutual fund with a return of 10% annually.

(a) What annual interest rate, when compounded monthly, gives an effective annual rate of 10%?
Assume that you invest \$1000. Then after a year at rate r compounded monthly, you have

$$A = 1000(1+\frac{r}{12})^{12}$$

If this had been invested with simple interest of 10% then you would have

$$A = 1000(.10)(1) = 1100$$

This is the amount of money you have actually accumulated by your compounding. Solving

$$\begin{aligned} 1100 &= 1000(1+\frac{r}{12})^{12} \\ 1.1 &= (1+\frac{r}{12})^{12} \\ \ln 1.1 &= 12\ln(1+\frac{r}{12}) \\ \frac{\ln 1.1}{12} &= 1+\frac{r}{12} \\ \frac{\ln 1.1}{12} - 1 &= \frac{r}{12} \\ 12\left(\frac{\ln 1.1}{12} - 1\right) &= r \\ r &\approx .09569 \end{aligned}$$

The rate is approximately 9.569%.

(b) What is Sue's monthly payment?

$$140,000 = R\left(\frac{1-\left(1+\frac{.06625}{12}\right)^{-360}}{\frac{.06625}{12}}\right)$$
$$R \approx 896.44$$

Her monthly payment is approximately \$896.44.

(c) If Sue invests the remainder of her \$1200 each month after the payment you found above, in a mutual fund with the interest rate found in (a), how much money will she have in the fund at the end of 30 years?

$$R = 1200 - 896.44 = 303.56$$
$$S = 303.56\left(\frac{\left(1+\frac{.09569}{12}\right)^{360}-1}{\frac{.09569}{12}}\right)$$
$$\approx \$626,200.88$$

(d) What is Joe's monthly payment?

$$140,000 = R\left(\frac{1-\left(1+\frac{.0625}{12}\right)^{-360}}{\frac{.0625}{12}}\right)$$
$$R \approx 1200.39$$

His payment is \$1200.39.

(e) If Joe invests the remainder of his \$1200 each month after the payment you found above, in a mutual fund with the interest rate found in (a), how much money will he have in the fund at the end of 30 years?

$$S = 1200\left(\frac{\left(1+\frac{.09569}{12}\right)^{15(12)}-1}{\frac{.09569}{12}}\right)$$
$$= \$478,134.14$$

He will have \$478,134.14

(f) Who is ahead at the end of 30 years and by how much? (Remember to calculate the interest they must pay, also)
In terms of retirement accounts Sue is ahead by

$$626,200.88 - 478,134.14 = \$148,066.74$$