Math 199
Summer II 2006
Quiz 4 - §2.4

Name: **Solutions**

Complete the following problems. Show all work to receive full credit.

1. Look at the polynomial $f(x) = 3(x-2)^2(x+3)(x-4)$.

   (a) What is the degree of the polynomial?
   The degree is $2+1+1=4$

   (b) What is the leading coefficient?
   The leading coefficient is 3.

   (c) What are the $x-$intercepts of the polynomial?
   These happen when $y=0$:

$$0 = 3(x-2)^2(x+3)(x-4)$$
$$x-2=0 \text{ or } x+3=0 \text{ or } x-4=0$$
$$x=2 \text{ or } x=-3 \text{ or } x=4$$

   (d) What is the $y-$intercept of the polynomial?
   This happens when $x=0$:

$$y = 3(x-2)^2(0+3)(0-4) = 3\cdot 4\cdot 3\cdot(-4) = -144$$

   (e) Draw a picture of the end behavior of the polynomial.
   Since this is an even degree polynomial with a positive leading coefficient, both ends point up.

2. Look at the rational function $g(x) = \dfrac{(x-2)(x+3)}{(x+4)(x+3)}$

   (a) What is the domain of the function? Label which domain problems give a vertical asymptote and which give holes in the graph.
   The domain is all x except for where the denominator is zero:

$$x+4 \neq 0 \text{ and } x+3 \neq 0$$
$$x \neq -4 \text{ and } x \neq -3$$

   $x=-4$ gives a vertical asymptote and $x=-3$ gives a hole in the graph (because it can cancel with the numerator).

   For all other parts of this problem, we look at the reduced form :$f(x) = \dfrac{x-2}{x+4}$

(b) What are the $x-$intercepts of the function?
These happen where the numerator is 0:

$$x - 2 = 0$$

$$x = 2$$

(c) What is the $y-$intercept of the function?
This happens when $x = 0$:

$$y = \frac{0-2}{0+4} = \frac{-2}{4} = -\frac{1}{2}$$

(d) What is the horizontal asymptote of the function?
Since the degrees of the top and the bottom are the same, the horizontal asymptote is the ratio of the leading coefficients, so $y = 1$.