

Here are some things to look at as you prepare for the final exam. The questions on this review sheet cover the material since the last exam, and a review of logarithms. The final exam is cumulative, so remember to look at Review for Exam 1 and Review for Exam 2 as you study. Also, remember to look at quizzes and homework problems, as well as the old exams. The actual exam will consist of a large number of true/false and multiple choice problems, but the problems below will help you understand the concepts covered on the exam. Try to do as many of these as you can without looking in your notes or book for guidance.

1. Simplify the following expressions:

(a)  $\ln e^3$

$$\ln e^3 = 3$$

since  $\ln x$  and  $e^x$  are inverse functions and un-do each other.

(b)  $10^{\log 7.4}$

$$10^{\log 7.4} = 7.4$$

since  $\log x$  and  $10^x$  are inverse functions and un-do each other.

(c)  $\log_8 16$

$$\log_8 16 = x$$

$$8^x = 16$$

$$(2^3)^x = 2^4$$

$$2^{3x} = 2^4$$

$$3x = 4$$

$$x = \frac{4}{3}$$

(d)  $\log_{25} 5$

$$\log_{25} 5 = x$$

$$25^x = 5$$

$$x = \frac{1}{2}$$

2. Write each expression as a single logarithm:

(a)  $\log 4k + \log 5k^3$

$$\log 4k + \log 5k^3 = \log (4k \cdot 5k^3) = \log 20k^4$$

(b)  $4 \ln x - 2 (\ln x^3 + 4 \ln x)$

$$\begin{aligned} 4 \ln x - 2 (\ln x^3 + 4 \ln x) &= \ln x^4 - 2 \ln x^3 - 8 \ln x \\ &= \ln x^4 - \ln(x^3)^2 - \ln x^8 \\ &= \ln x^4 - \ln x^6 - \ln x^8 \\ &= \ln \left( \frac{x^4}{x^6} \right) - \ln x^8 \\ &= \ln \left( \frac{1}{x^2} \right) - \ln x^8 \\ &= \ln \left( \frac{\frac{1}{x^2}}{x^8} \right) \\ &= \ln \left( \frac{1}{x^2} \cdot \frac{1}{x^8} \right) \\ &= \ln \left( \frac{1}{x^{10}} \right) \end{aligned}$$

3. Solve each equation:

(a)  $\ln(m + 3) - \ln m = \ln 2$

$$\ln(m + 3) - \ln m = \ln 2$$

$$\ln \left( \frac{m + 3}{m} \right) = \ln 2$$

$$\frac{m + 3}{m} = 2$$

$$m + 3 = 2m$$

$$m = 3$$

(b)  $2 \ln(y + 1) = \ln(y^2 - 1) + \ln 5$

$$2 \ln(y + 1) = \ln(y^2 - 1) + \ln 5$$

$$\ln(y + 1)^2 = \ln((y^2 - 1)(5))$$

$$(y + 1)^2 = 5(y^2 - 1)$$

$$y^2 + 2y + 1 = 5y^2 - 5$$

$$0 = 4y^2 - 2y - 6$$

$$0 = 2(2y^2 - y - 3)$$

$$0 = 2(2y - 3)(y + 1)$$

$$y = \frac{3}{2} \text{ or } y = -1$$

Since  $y = -1$  is not in the domain of  $\ln(y + 1)$ , the only solution is  $y = \frac{3}{2}$

(c)  $\log(m + 2) = 1$

$$\log(m + 2) = 1$$

$$m + 2 = 10^1$$

$$m + 2 = 10$$

$$m = 8$$

(d)  $\log_2(3k - 2) = 4$

$$\log_2(3k - 2) = 4$$

$$2^4 = 3k - 2$$

$$3k - 2 = 16$$

$$3k = 18$$

$$k = \frac{18}{3}$$

(e)  $\log_5\left(\frac{5z}{z-2}\right) = 2$

$$\log_5\left(\frac{5z}{z-2}\right) = 2$$

$$5^2 = \frac{5z}{z-2}$$

$$25(z - 2) = 5z$$

$$25z - 50 = 5z$$

$$20z = 50$$

$$z = \frac{50}{20} = \frac{5}{2}$$

(f)  $\log_2 r + \log_2(r - 2) = 3$

$$\log_2 r + \log_2(r - 2) = 3$$

$$\log_2(r(r - 2)) = 3$$

$$r(r - 2) = 2^3$$

$$r^2 - 2r = 8$$

$$r^2 - 2r - 8 = 0$$

$$(r - 4)(r + 2) = 0$$

$$r = 4 \text{ or } r = -2$$

Since  $-2$  is not in the domain of  $\log_2 r$ , the only solution is  $r = 4$ .

$$(g) \quad 2^{3x} = \frac{1}{8}$$

$$2^{3x} = \frac{1}{8}$$

$$2^{3x} = 2^{-3}$$

$$3x = -3$$

$$x = -1$$

$$(h) \quad \left(\frac{9}{16}\right)^x = \frac{3}{4}$$

$$\left(\frac{9}{16}\right)^x = \frac{3}{4}$$

$$\left[\left(\frac{3}{4}\right)\right]^x = \frac{3}{4}$$

$$\left(\frac{3}{4}\right)^{2x} = \left(\frac{3}{4}\right)^1$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$(i) \quad 9^{2y-1} = 27^y$$

$$9^{2y-1} = 27^y$$

$$(3^2)^{2y-1} = (3^3)^y$$

$$3^{4y-2} = 3^{3y}$$

$$4y - 2 = 3y$$

$$y = 2$$

$$(j) \quad 8^p = 19$$

$$8^p = 19$$

$$\ln 8^p = \ln 19$$

$$p \ln 8 = \ln 19$$

$$p = \frac{\ln 19}{\ln 8}$$

(k)  $6^{2-m} = 2^{3m+1}$

$$\begin{aligned}
 6^{2-m} &= 2^{3m+1} \\
 \ln 6^{2-m} &= \ln 2^{3m+1} \\
 (2-m) \ln 6 &= (2m+1) \ln 2 \\
 2 \ln 6 - m \ln 6 &= 3m \ln 2 + \ln 2 \\
 2 \ln 6 - \ln 2 &= 3m \ln 2 + m \ln 6 \\
 2 \ln 6 - \ln 2 &= m(3 \ln 2 + \ln 6) \\
 m &= \frac{2 \ln 6 - \ln 2}{3 \ln 2 + \ln 6}
 \end{aligned}$$

(l)  $2 \cdot 15^{-k} = 18$

$$\begin{aligned}
 2 \cdot 15^{-k} &= 18 \\
 15^{-k} &= 9 \\
 \ln 15^{-k} &= \ln 9 \\
 -k \ln 15 &= \ln 9 \\
 k &= -\frac{\ln 9}{\ln 15}
 \end{aligned}$$

4. Know the following formulas. They will **NOT** be provided on the exam.

	<b>Interest</b>	<b>Future Value</b>	<b>Present Value</b>
Simple Interest	$I = Prt$	$A = P(1 + rt)$	$P = \frac{A}{1 + rt}$
Compound Interest	$I = A - P$	$A = P(1 + i)^n$	$P = \frac{A}{(1 + i)^n}$
Continuous Interest	$I = A - P$	$A = Pe^{rt}$	$P = \frac{A}{e^{rt}}$
Ordinary Annuity		$S = R \left( \frac{1 + i)^n - 1}{i} \right)$	$P = R \left( \frac{1 - (1 + i)^{-n}}{i} \right)$

5. Find the simple interest for an investment of \$4902 at 9/5% for 11 months.

$$\begin{aligned} I &= Prt \\ &= 4902(.095) \left( \frac{11}{12} \right) \\ &\approx \$426.88 \end{aligned}$$

6. Find the future value of an investment of \$3478 at 7.4% for 88 days (assume 365 days in a year).

$$\begin{aligned} I &= Prt \\ &= 3478 (.074) \left( \frac{88}{365} \right) \\ &\approx \$62.05 \end{aligned}$$

7. Find the present value of the future amount \$459.57 if the money is invested at a simple interest rate of 8.5% for 7 months.

$$\begin{aligned} A &= P(1 + rt) \\ 459.57 &= P \left( 1 + .085 \left( \frac{7}{12} \right) \right) \\ P &= \frac{459.57}{1 + .085 \left( \frac{7}{12} \right)} \\ &\approx \$437.86 \end{aligned}$$

8. Find the amount of money in an account in which the initial deposit is \$2800 earning 6% interest compounded annually after 10 years.

$$\begin{aligned} A &= P(1 + i)^n \\ &= 2800(1 + .06)^{10} \\ &= 2800(1.06)^{10} \\ &\approx \$5014.37 \end{aligned}$$

9. Find the present value of the future amount \$42,000 if the money is deposited in an account paying 12% compounded monthly for 7 years.

$$\begin{aligned} A &= P(1 + i)^n \\ 42,000 &= P \left( 1 + \frac{.12}{12} \right)^{84} \\ P &= \frac{42,000}{(1.01)^{84}} \\ &\approx \$18,207.65 \end{aligned}$$

10. Find the future value of the annuity where \$1288 is deposited at the end of each year for 14 years, where the money earns 8% compounded annually.

$$A = 1288 \left( \frac{(1 + .08)^{14} - 1}{.08} \right)$$

$$\approx \$31,188.82$$

11. Find the future value of the annuity where \$233 is deposited at the end of each month for 4 years and the money earns 12% compounded monthly.

$$A = 233 \left( \frac{(1 + \frac{.12}{12})^{48} - 1}{\frac{.12}{12}} \right)$$

$$\approx \$14,264.87$$

12. Find the present value of the ordinary annuity where payments of \$850 are made annually for 4 years at 5% compounded annually.

$$PV = P \left( \frac{1 - (1 + i)^{-n}}{i} \right)$$

$$PV = 850 \left( \frac{1 - (1 + .05)^{-4}}{.05} \right)$$

$$\approx \$3014.06$$

13. Find the present value of the ordinary annuity where payments of \$4210 are made semiannually for 8 years at 8.6% compounded semiannually.

$$PV = P \left( \frac{1 - (1 + i)^{-n}}{i} \right)$$

$$= 4210 \left( \frac{1 - (1 + \frac{.08}{2})^{-2 \cdot 8}}{\frac{.08}{2}} \right)$$

$$\approx \$??$$

14. Find the amount of the payment necessary to amortize a loan of \$32,000 at 9.4% compounded quarterly to be repaid in 10 quarterly payments.

$$PV = P \left( \frac{1 - (1 + i)^{-n}}{i} \right)$$

$$32,000 = P \left( \frac{1 - (1 + \frac{.094}{4})^{-10}}{\frac{.094}{4}} \right)$$

$$P = \frac{32,000}{\left( \frac{1 - (1 + \frac{.094}{4})^{-10}}{\frac{.094}{4}} \right)}$$

$$\approx \$3628.00$$

15. Find the monthly house payment for a mortgage of \$56,890 at 10.74% for 25 years.

$$\begin{aligned}PV &= P \left( \frac{1 - (1 + i)^{-n}}{i} \right) \\56,890 &= P \left( \frac{1 - \left(1 + \frac{.1074}{12}\right)^{-300}}{\frac{.1074}{12}} \right) \\P &= \frac{56,890}{\left( \frac{1 - \left(1 + \frac{.1074}{12}\right)^{-300}}{\frac{.1074}{12}} \right)} \\&\approx \$546.93\end{aligned}$$

16. A firm of attorneys deposits \$15,000 of profit-sharing money in an account at 6% compounded semiannually for 7.5 years. Find the amount of interest earned.

$$\begin{aligned}A &= 15,000 \left( 1 + \frac{.06}{2} \right)^{15} \\&= \$23,369.51 \\I &= A - P \\&= \$23,369.51 - \$15,000 \\&= \$8369.51\end{aligned}$$

17. According to a financial web site, Bank A paid 6.9% interest compounded quarterly on a one year CD, and Bank B paid 6.88% compounded monthly. What are the effective rates for the two CDs and which bank pays a higher effective rate?

Assume a deposit of \$100. For Bank A, after one year, the account will have:

$$\begin{aligned}A &= 100 \left( 1 + \frac{.069}{4} \right) \\&= 107.08\end{aligned}$$

Therefore the effective rate is 7.08%

For Bank B, after one year, the account will have:

$$\begin{aligned}A &= 100 \left( 1 + \frac{.0688}{12} \right) \\&= 107.10\end{aligned}$$

Therefore the effective rate is 7.10%, and Bank B has a higher effective rate.

18. Each year a firm must set aside enough funds to provide employee retirement benefits of \$52,000 in 20 years. If the firm can invest money at 7.5% compounded monthly, what amount must be invested at the end of each month for this purpose?

$$A = P \left( \frac{(1+i)^n - 1}{i} \right)$$

$$52,000 = P \left( \frac{\left(1 + \frac{.075}{12}\right)^{240} - 1}{\frac{.075}{12}} \right)$$

$$P = \frac{52,000}{\left( \frac{\left(1 + \frac{.075}{12}\right)^{240} - 1}{\frac{.075}{12}} \right)}$$

$$\approx \$93.91$$

The firm should invest \$93.91 monthly.

19. In 3 years, Mary must pay a pledge of \$7500 to her favorite charity. What lump sum can she deposit today, at 10% compounded semiannually, so that she will have enough to pay the pledge?

$$A = P(1+i)^n$$

$$7500 = P \left( 1 + \frac{.10}{2} \right)^6$$

$$P = \frac{7500}{\left( 1 + \frac{.10}{2} \right)^6}$$

$$\approx \$5596.62$$

20. Use algebraic techniques (show all steps) to solve the following systems of equations:

(a)  $-5x - 3y = 4$

$2x + y = -3$

Use the second equation to solve for  $y$ .

$$y = -3 - 2x$$

Substitute this into the first equation:

$$-5x - 3y = 4$$

$$-5x - 3(-3 - 2x) = 4$$

$$-5x + 9 + 6x = 4$$

$$x = -5$$

Plug this into the equation to solve for  $x$ :

$$y = -3 - 2x$$

$$\begin{aligned}
 y &= -3 - 2(-5) \\
 &= -3 + 10 \\
 &= 7
 \end{aligned}$$

Therefore the solution is  $x = -5$ ,  $y = 7$

(b)  $3x + y - z = 13$

$$x + 2z = 9$$

$$-3x - y + 2z = 9$$

Add the first and third equations:

$$\begin{array}{r}
 3x + y - z = 13 \\
 -3x - y + 2z = 9 \\
 \hline
 z = 22
 \end{array}$$

Plug this into the second equation:

$$x + 2z = 9$$

$$x + 2(22) = 9$$

$$x + 44 = 9$$

$$x = -35$$

Plug both of these into the first equation:

$$\begin{aligned}
 3x + y - z &= 13 \\
 3(-35) + y - 22 &= 13 \\
 -105 + y - 22 &= 13 \\
 y - 127 &= 13 \\
 y &= 140
 \end{aligned}$$

The solution is  $x = -35$ ,  $y = 140$ ,  $z = 22$ .

21. Gretchen Schmidt plans to buy shares of two stocks. One costs \$32 per share and pays dividends of \$1.20 per share. The other costs \$23 per share and pays dividends of \$1.40 per share. She has \$10,100 to spend and wants to earn dividends of \$540. How many shares of each stock should she buy?

Let  $x$  = the number of shares of the first stock

$y$  = the number of shares of the second stock

$$32x + 23y = 10,100$$

$$1.2x + 1.4y = 540$$

Multiply equation 1 by 1.2 and equation 2 by -32 and add the results:

$$\begin{aligned}
38.4x + 27.6y &= 12,120 \\
-38.4x - 44.8y &= -17,280 \\
\hline
-17.2y &= -5160 \\
y &= 300
\end{aligned}$$

Plug this into equation 2:

$$\begin{aligned}
1.2x + 1.4y &= 540 \\
1.2x + 1.4(300) &= 540 \\
1.2x + 420 &= 540 \\
1.2x &= 120 \\
x &= 100
\end{aligned}$$

She should buy 100 shares of the first stock and 300 shares of the second stock.

22. Joyce Pluth has money in two investment funds. Last year the first fund paid a dividend of 8% and the second dividend of 2% and Joyce received a total of \$780. This year the first fund paid a 10% dividend and the second only 1% and Joyce received \$810. How much does she have invested in each fund?

Let  $x$  = the amount invested in the first fund

$y$  = the amount invested in the second fund

$$\begin{aligned}
.08x + .02y &= 780 \\
.1x + .01y &= 810
\end{aligned}$$

Multiply the second equation by -2 and add to the first equation:

$$\begin{aligned}
.08x + .02y &= 780 \\
-.2x - .02y &= -1620 \\
\hline
-.12x &= -840 \\
x &= 7000
\end{aligned}$$

Substitute this into the first equation:

$$\begin{aligned}
.08x + .02y &= 780 \\
.08(7000) + .02y &= 780 \\
560 + .02y &= 780 \\
.02y &= 220 \\
y &= 11,000
\end{aligned}$$

Joyce has invested \$7000 in the first fund and \$11,000 in the second fund.

23. Shirley Cicero has \$16,000 invested in Boeing and GE stock. The Boeing stock currently sells for \$30 per share and the GE stock for \$70 per share. If GE stock triples its value and the Boeing stock goes up 50%, her stock will be worth \$34,500. How many shares of each stock does she own?

Let  $x$  = number of shares of Boeing stock.

Let  $y$  = number of shares of GE stock.

$$30x + 70y = 16,000$$

If the price of the GE stock triples then it will be worth  $70 \cdot 3 = \$210$  per share. If the price of the Boeing stock increases by 50%, it will be worth  $30 \cdot 1.5 = \$45$  per share. Therefore we also get the following equation:

$$45x + 210y = 34500$$

Multiply the first equation by  $-3$ :

$$-90x - 210y = -48,000$$

$$\underline{45x + 210y = 34500}$$

$$-45x = -13,500$$

$$x = 300$$

Plugging back in, we get:

$$45(300) + 210y = 34500$$

$$13,500 + 210y = 34500$$

$$210y = 21000$$

$$y = 100$$

Therefore she has 300 shares of Boeing stock and 100 shares of GE stock.

24. Pretzels cost \$3 per pound, dried fruit \$4 per pound and nuts \$8 per pound. How many pounds of each should be used to produce 140 pounds of trail mix costed \$6 per pound in which there are twice as many pretzels (by weight) as dried fruit?

Let  $p$  = number of pounds of pretzels

$f$  = number of pounds of dried fruit

$n$  = number of pounds of nuts

$$p = 2f \Rightarrow 2f - p = 0$$

$$p + f + n = 140$$

$$3p + 4f + 8n = 140(6)$$

Using the first equation to rewrite the other two:

$$p = 2f$$

$$2f + f + n = 140 \Rightarrow 3f + n = 140$$

$$3(2f) + 4f + 8n = 840 \Rightarrow 10f + 8n = 840$$

Solving the second equation for  $n$ , we get

$$n = 140 - 3f$$

and so the last equation becomes

$$10f + 8(140 - 3f) = 840$$

$$10f + 1120 - 24f = 840$$

$$-14f = -280$$

$$f = 20$$

Substituting back:

$$n = 140 - 3f = 140 - 3(20) = 140 - 60 = 80$$

$$p = 2f = 2(20) = 40$$

Therefore, they should use 40 pounds of pretzels, 20 pounds of dried fruit and 80 pounds of nuts.

25. You are given \$144 in one, five, and ten dollar bills. There are two more ten dollar bills than five dollar bills. If you have a total of 35 bills, how many bills of each type are there?

Let  $x$  = number of one dollar bills

$y$  = number of five dollar bills

$z$  = number of ten dollar bills

We get the equations:

$$x + y + z = 35$$

$$x + 5y + 10z = 144$$

$$z = y + 2 \Rightarrow z - y = 2$$

We can use the third equation to rewrite the first two:

$$x + y + y + 2 = 35 \Rightarrow x + 2y = 33$$

$$x + 5y + 10(y + 2) = 144 \Rightarrow x + 5y + 10y + 20 = 144 \Rightarrow x + 15y = 124$$

$$x = y + 2$$

Solve the first equation for  $x$ :  $x = 33 - 2y$  and substitute into the second equation:

$$33 - 2y + 15y = 91$$

$$13y = 91$$

$$y = 7$$

$$x = 33 - 2y = 33 - 2(7) = 19$$

$$z = y + 2 = 7 + 2 = 9$$

therefore there are 19 ones, 7 fives, and 9 tens.

26. Use matrices and your calculator to solve the following systems;

$$\begin{aligned} \text{(a)} \quad & 4x - y - 2z = 4 \\ & x - y - \frac{1}{2}z = 1 \\ & 2x - y - z = 8 \end{aligned}$$

$$\left( \begin{array}{ccc|c} 4 & -1 & -2 & 4 \\ 1 & -1 & -\frac{1}{2} & 1 \\ 2 & -1 & -1 & 8 \end{array} \right)$$

By calculator:

$$\left( \begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right)$$

Because this last row says that  $0 = 1$  the system is inconsistent and it has no solutions.

$$\begin{aligned} \text{(b)} \quad & x - z = -3 \\ & y + z = 6 \\ & 2x - 3z = -9 \end{aligned}$$

$$\left( \begin{array}{ccc|c} 1 & 0 & -1 & -3 \\ 0 & 1 & 1 & 6 \\ 2 & 0 & -3 & -9 \end{array} \right)$$

By calculator:

$$\left( \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{array} \right)$$

The solution is  $(0, 3, 3)$ , i.e.  $x = 0$ ,  $y = 3$ , and  $z = 3$

$$\begin{aligned} \text{(c)} \quad & x - 2y + 3z = 4 \\ & 2x + y - 4z = 3 \\ & -3x + 4y - z = -2 \end{aligned}$$

$$\left( \begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 2 & 1 & -4 & 3 \\ -3 & 4 & -1 & -2 \end{array} \right)$$

$$\left( \begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array} \right)$$

The solution is  $x = 4$ ,  $y = 3$ ,  $z = 2$ .

27. In deciding whether to set up a new manufacturing plant, company analysts have decided that a linear function is a reasonable estimation for the total cost  $C(x)$  in dollars to produce  $x$  items. They estimate the cost to produce 10,000 items as \$547,500 and the cost to produce 50,000 items as \$737,500.

- (a) Find a formula for  $C(x)$ .

We know two points on the cost function:

$$C(10,000) = 547,500$$

$$C(50,000) = 737,500$$

Since the cost function is linear we can use these two points to find the marginal cost, which will be the slope of the linear function:

$$m = \frac{737,500 - 547,500}{50,000 - 10,000} = \frac{190,000}{40,000} = 4.75$$

Therefore, we have the cost function looks like:

$$C(x) = 4.75x + f$$

where  $f$  is the fixed costs. Pick a point to plug into this equation to solve for  $f$ :

$$577,500 = 4.75(10,000) + f$$

$$547,500 = 47,500 + f$$

$$f = 500,000$$

Therefore the linear cost function is

$$C(x) = 4.75x + 500,000$$

- (b) Find the total cost to produce 100,000 items.

$$C(100,000) = 4.75(100,000) + 500,000 = 975,000$$

- (c) Find the marginal cost of the items produced in this plant.

This is the slope of the cost function, so 4.75 per item.

28. Colleen Davis owns a factory that manufactures souvenir key chains. her weekly profit (in hundred of dollars) is given by  $P(x) = -2x^2 + 60x - 120$ , where  $x$  is the number of cases of key chains sold.

- (a) What is the largest number of cases she can sell and still make a profit?

$$P(x) = -2x^2 + 60x - 120$$

We want  $P(x) > 0$ :

$$-2x^2 + 60x - 120 > 0$$

$$-2(x^2 - 30x + 60) > 0$$

$$x^2 - 30x + 60 < 0$$

Think:

$$\begin{aligned}x^2 - 30x + 60 &= 0 \\x &= \frac{30 \pm \sqrt{900 - 4(1)(60)}}{2(1)} \\&= \frac{30 \pm \sqrt{660}}{2} \\&= 15 \pm \sqrt{165}\end{aligned}$$

The highest input where the expression  $x^2 - 30x + 60$  is negative is

$$x = 15 + \sqrt{165} \approx 40.69$$

Therefore the largest number of cases that she can sell and still make a profit is 40

(b) How many cases should she make and sell in order to maximize her profits?

This occurs at the vertex of the function:

$$\begin{aligned}P(x) &= -2x^2 + 60x - 120 \\&= -2(x^2 - 30x) - 120 \\&= -2(x^2 - 30x + 225 - 225) - 120 \\&= -2((x - 15)^2 - 225) - 120 \\&= -2(x - 15)^2 + 450 - 120 \\&= -2(x - 15)^2 + 330\end{aligned}$$

Therefore she should sell 15 cases to maximize her profits.

(c) What is the maximum profit she could earn?

The maximum profit she could earn is \$330.

29. The profit (in millions of dollars) from the sale of  $x$  million units of Blue Glue is given by  $p = .7x - 25.5$ . The cost is given by  $c = .9x + 25.5$ .

(a) Find the revenue equation.

$$\begin{aligned}p &= r - c \\7x - 25.5 &= r - (.9x + 25.5) \\7x - 25.5 &= r - .9x - 25.5 \\1.6x - 25.5 + 25.5 &= r \\r(x) &= 1.6x\end{aligned}$$

(b) What is the revenue from selling 10 million units?

$$r(10) = 1.6(10) = 16\text{million dollars}$$

- (c) What is the break even point?  
This is where profit is 0.

$$.7x - 25.5 = 0$$

$$.7x = 25.5$$

$$x \approx 36.42857143$$

or approximately 36,428,571 units