Math 199
Fall, 2005
Review for Exam 2

Here are some things to look at as you prepare for the exam. Remember to look at quizzes and homework problems also. The actual exam will consist of a large number of true/false and multiple choice problems, but the problems below will help you understand the concepts covered on the exam. Try to do as many of these as you can without looking in your notes or book for guidance.

The second exam will be on Thursday November 3, and will cover material in Sections 2.4, 3.3, and Chapter 5.

1. Graph the following polynomial functions:

   (a) $f(x) = x^3 - x$
   This has the end behavior of an odd degree polynomial with positive leading coefficient, and has $x-$intercepts at $0, 1, -1$.

   (b) $f(x) = x(x-2)(x+3)$
   This has the end behavior of an odd degree polynomial with positive leading coefficient, and has $x-$intercepts at $0, 2, -3$

   (c) $f(x) = x^4 - 7x^2 - 8$
   This has the end behavior of an even degree polynomial with positive leading coefficient, and has $x-$intercepts at $\sqrt{8}, -\sqrt{8}$.

2. For the following functions, do the following:

   - Find the vertical asymptotes.
   - Find the horizontal asymptote
   - Find the $x-$intercepts
   - Graph the function

   (a) $f(x) = \dfrac{1}{x-3}$

   - Find the vertical asymptotes.
     These happen where the denominator is 0, so at $x = 3$
   - Find the horizontal asymptote
     Since the degree of the denominator is larger than the degree of the numerator, the horizontal asymptote is $y = 0$
   - Find the $x-$intercepts
     These happen where the numerator is 0, which never happens for this function.

(b) $g(x) = \dfrac{5x-2}{4x^2-4x+3}$

- Find the vertical asymptotes.
  These happen where the denominator is zero, so where

$$4x^2 - 4x + 3 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{4 \pm \sqrt{16-4(4)(3)}}{2(4)}$$

$$= \frac{4 \pm \sqrt{16-48}}{8}$$

  Since we cannot take the square root of a negative number, there are no vertical asymptotes for this function.
- Find the horizontal asymptote
  Since the degree of the denominator is larger than the degree of the numerator, the horizontal asymptote is $y = 0$
- Find the $x-$intercepts
  These happen where the numerator is 0, so

$$5x - 2 = 0$$

$$5x = 2$$

$$x = \frac{2}{5}$$

(c) $h(x) = \dfrac{x^2-4}{x+2}$

First let's factor the numerator and denominator of the function:

$$h(x) = \frac{x^2-4}{x+2} = \frac{(x-2)(x+2)}{x+2}$$

- Find the vertical asymptotes.
  These happen where the denominator is zero, as long as these problems don't cancel with the numerator, so we check where

$$x + 2 = 0$$

$$x = -2$$

  However, this problem cancels with the numerator, and so $x = -2$ is a place where we have a hole in the graph, and there are no vertical asymptotes
  In fact, this function looks just like the function $f(x) = x - 2$ except for the hole at $x = -2$
- Find the horizontal asymptote
  There is no horizontal asymptote, since this function looks like a line.
- Find the $x-$intercepts
  This is where the numerator is zero, so where

$$x - 2 = 0$$

$$x = 2$$

- Graph the function
  the graph looks like a line with $y-$intercept -2 and slope 1

3. A cost-benefit curve for pollution control is given by

$$y = \frac{9.2x}{106 - x}$$

   where y is the cost in thousands of dollars of removing x percent of a specific industrial pollutant.

   (a) Find y if $x = 50$
   if $x = 50$ then

$$y = \frac{9.2(50)}{106 - 50} = 8.2 \text{ thousand dollars} = \$8200$$

   (b) Find y if $x = 98$
   If $x = 98$, then

$$y = \frac{9.2(98)}{106 - 98} = 112.7 \text{ thousand dollars} = \$112,700$$

   (c) What percent of the pollutant can be removed for \$22,000?
   This means $y = \$22$, and so we have:

$$22 = \frac{9.2x}{106 - x}$$

$$22(106 - x) = 9.2x$$

$$2332 - 22x = 9.2x$$

$$2332 = 31.2x$$

$$x \approx 74.7$$

   So about 75% of pollutants can be removed for \$22,000.

4. If world population continues to grow as expected, the population (in billions) in year t will be given by the function

$$P(t) = 4.834\,(1.01)^{(t-1980)}$$

   .

   (a) Estimate the world population in the year 2005

$$P(2005) = 4.834\,(1.01)^{(2005-1980)} = 4.834(1.01^{25}) \approx 6.2$$

   In 2005, the population will be about 6.3 billion

   (b) Estimate the world population in the year 2010

$$P(2010) = 4.834\,(1.01)^{(2010-1980)} = 4.834(1.01^{30}) \approx 6.5$$

   In 2010, the population will be about 6.5 billion.

(c) Estimate the world population in the year 2030

$$P(2030) = 4.834\,(1.01)^{(2030-1980)} = 4.834(1.01^{50}) \approx 8.0$$

In 2030, the population will be about 8 billion.

5. The scrap value of a machine is the value of the machine at the end of its useful life. By one method of calculating scrap value, where it is assumed a constant percentage of value is lost annually, the scrap value S is given by

$$S = C(1-r)^n$$

where C is the original cost, n is the useful life of the machine in years, and r is the constant annual percentage of value lost. Find the scrap value for each of the following machines:

(a) Original Cost \$54,000; life 8 years; annual rate of loss 12%

$$S = 54,000(1-.12)^8 = 54,000(.88)^8 \approx \$19,420.26$$

(b) Original Cost \$178,000; life 11 years; annual rate of loss 14%

$$S = 178,000(1-.14)^{11} \approx \$33,876.85$$

6. The US Census Bureau predicts that the African-American population will increase from 35.3 million in 2000 to 59.2 million in 2050.

(a) Find a model for this data in which $t = 0$ corresponds to 2000.
Let $f(t)$ be the African American population in millions in year t after 2000.

$$f(t) = y_0 b^t$$
$$f(0) = 35.3 = y_0 b^0$$
$$35.3 = y_0$$

$$f(t) = 35.3b^t$$

$$f(50) = 59.2 = 35.3b^{50}$$
$$\frac{59.2}{35.3} = b^{50}$$
$$\left(\frac{59.2}{35.3}\right)^{\frac{1}{50}} = b$$
$$b \approx 1.010394$$

The model is

$$f(t) = 35.3(1.010394)^t$$

(b) What is the projected African-American population in 2004? in 2030?

In 2004

$$f(4) = 35.3(1.010394)^4 \approx 36.8$$

the population will be about 36.8 million

In 2030

$$f(30) = 35.3(1.010394)^{30} \approx 48.1$$

the projected African-American population will be about 48.1 million

(c) Estimate the year in which the African American population will reach 55 million.

$$55 = 35.3(1.010394)^t$$

By trial and error, this should occur in 2043.

7. Newton's Law of Cooling says that the rate at which a body cools is proportional to the difference in temperature between the body and an environment into which it is introduced. The temperature $F(t)$ of the body at time t after being introduced into an environment have constant temperature T_0 is

$$F(t) = T_0 + Cb^t$$

where C and b are constants.

Boiling water, at 100°C, is placed in a freezer at 0° C. The temperature of the water is 50° C after 24 minutes. Find the temperature of the water after 96 minutes.

$$F(t) = T_0 + Cb^t$$
$$T_0 = 0$$
$$F(0) = 100 = 0 + Cb^0$$
$$100 = C(1)$$
$$C = 100$$

$$F(t) = 0 + 100b^t$$
$$F(t) = 100b^t$$

At time 24,

$$50 = 100b^{24}$$
$$\frac{1}{2} = b^{24}$$
$$\left(\frac{1}{2}\right)^{\frac{1}{24}} = b$$
$$b \approx .9715$$

$$F(t) = 100(.9715)^t$$
$$F(96) = 100(.9715)^{96} \approx 6.25°C$$

8. Simplify the following expressions:

(a) $\ln e^3$

$$= 3$$

(b) $10^{\log 7.4}$

$$= 7.4$$

(c) $\log_8 16$

$$\begin{aligned} \log_8 16 &= y \\ 8^y &= 16 \\ (2^3)^y &= 2^4 \\ 2^{3y} &= 2^4 \\ 3y &= 4 \\ y &= \frac{4}{3} \\ \log_8 16 &= \frac{4}{3} \end{aligned}$$

(d) $\log_{25} 5$

9. Write each expression as a single logarithm:

(a) $\log 4k + \log 5k^3$

$$\begin{aligned} &= \log\big((4k)(5k^3)\big) \\ &= \log 20k^4 \end{aligned}$$

(b) $4\ln x - 2\left(\ln x^3 + 4\ln x\right)$

$$\begin{aligned} &= \ln x^4 - 2\ln x^3 - 8\ln x \\ &= \ln x^4 - \ln x^6 - \ln x^8 \\ &= \ln \frac{x^4}{x^6} - \ln x^8 \\ &= \ln \frac{1}{x^2} - \ln x^8 \\ &= \ln\left(\frac{x^{-2}}{x^8}\right) \\ &= \ln x^{-10} \end{aligned}$$

10. Solve each equation:

(a) $\ln(m+3) - \ln m = \ln 2$

$$\begin{aligned} \ln \frac{m+3}{m} &= \ln 2 \\ e^{\ln \frac{m+3}{m}} &= e^{\ln 2} \\ \frac{m+3}{m} &= 2 \\ m+3 &= 2m \\ m &= 3 \end{aligned}$$

(b) $2\ln(y+1) = \ln(y^2-1) + \ln 5$

$$\begin{aligned} \ln(y+1)^2 &= \ln(y^2-1) + \ln 5 \\ \ln(y+1)^2 - \ln(y^2+1) &= \ln 5 \\ \ln \frac{(y+1)^2}{y^2+1} &= \ln 5 \\ e^{\ln \frac{(y+1)^2}{y^2+1}} &= e^{\ln 5} \\ \frac{(y+1)^2}{y^2+1} &= 5 \\ y^2+2y+1 &= 5y^2+5 \\ 0 &= 4y^2-2y+4 \\ 0 &= 2(2y^2-y+2) \\ 0 &= 2y^2-y+2 \\ y &= \frac{1 \pm \sqrt{1-4(2)(2)}}{4} \end{aligned}$$

Therefore there are no solutions (the number under the radical is negative).

(c) $\log(m+2) = 1$

$$\begin{aligned} m+2 &= 10^1 \\ m+2 &= 10 \\ m &= 8 \end{aligned}$$

(d) $\log_2(3k-2) = 4$

$$\begin{aligned} 2^4 &= 3k-2 \\ 16 &= 3k-2 \\ 18 &= 3k \\ k &= \frac{18}{3} \end{aligned}$$

(e) $\log_5\left(\dfrac{5z}{z-2}\right) = 2$

$$\begin{aligned} 5^2 &= \frac{5z}{z-2} \\ 25 &= \frac{5z}{z-2} \\ 25(z-2) &= 5z \\ 25z - 50 &= 5z \\ 20z &= 50 \end{aligned}$$

(f) $\log_2 r + \log_2(r-2) = 3$

$$\begin{aligned} \log_2\left(r(r-2)\right) &= 3 \\ 2^3 &= r(r-2) \\ 8 &= r^2 - 2r \\ r^2 - 2r - 8 &= 0 \\ (r-4)(r+2) &= 0 \\ r = 4 \text{ or } r &= -2 \end{aligned}$$

Notice that $r = -2$ is not in the domain of the original equation, so this cannot be a solution. Therefore the only solution is $r = 4$.

(g) $2^{3x} = \dfrac{1}{8}$

$$\begin{aligned} 2^{3x} &= 2^{-3} \\ 3x &= -3 \\ x &= -1 \end{aligned}$$

(h) $\left(\dfrac{9}{16}\right)^x = \dfrac{3}{4}$

$$\begin{aligned} \left(\left(\frac{3}{4}\right)^2\right)^x &= \frac{3}{4} \\ \left(\frac{3}{4}\right)^{2x} &= \frac{3}{4} \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

(i) $9^{2y-1} = 27^y$

$$\begin{aligned}
\left(3^2\right)^{2y-1} &= \left(3^3\right)^y \\
3^{2(2y-1)} &= 3^{3y} \\
2(2y-1) &= 3y \\
4y - 2 &= 3y \\
y - 2 &= 0 \\
y &= 2
\end{aligned}$$

(j) $8^p = 19$

$$\begin{aligned}
\ln 8^p &= \ln 19 \\
p \ln 8 &= \ln 19 \\
p &= \frac{\ln 19}{\ln 8}
\end{aligned}$$

(k) $6^{2-m} = 2^{3m+1}$

$$\begin{aligned}
\ln 6^{2-m} &= \ln 2^{3m+1} \\
(2-m)\ln 6 &= (3m+1)\ln 2 \\
2\ln 6 - m\ln 6 &= 3m\ln 2 + \ln 2 \\
2\ln 6 - \ln 2 &= 3m\ln 2 + m\ln 6 \\
2\ln 6 - \ln 2 &= m\left(3\ln 2 + \ln 6\right) \\
m &= \frac{2\ln 6 - \ln 2}{3\ln 2 + \ln 6}
\end{aligned}$$

(l) $2 \cdot 15^{-k} = 18$

$$\begin{aligned}
25^{-k} &= 9 \\
\ln 25^{-k} &= \ln 9 \\
-k\ln 25 &= \ln 9 \\
k &= \frac{\ln 9}{-\ln 25}
\end{aligned}$$

11. Use matrices and your calculator to solve the following systems;

(a) $-5x - 3y = 4$
$2x + y = -3$
This can be done by hand, too:
Use the second equation to solve for y.

$$y = -3 - 2x$$

Substitute this into the first equation:

$$-5x - 3y = 4$$

$$-5x - 3(-3 - 2x) = 4$$
$$-5x + 9 + 6x = 4$$
$$x = -5$$

Plug this into the equation to solve for x:

$$y = -3 - 2x$$

$$y = -3 - 2(-5)$$
$$= -3 + 10$$
$$= 7$$

Therefore the solution is $x = -5$, $y = 7$

(b) $3x + y - z = 13$
$x + 2z = 9$
$-3x - y + 2z = 9$
Also, done by hand, this is:
Add the first and third equations:

$$3x + y - z = 13$$

$$\underline{-3x - y + 2z = 9}$$
$$z = 22$$

Plug this into the second equation:

$$x + 2z = 9$$
$$x + 2(22) = 9$$
$$x + 44 = 9$$
$$x = -35$$

Plug both of these into the first equation:

$$3x + y - z = 13$$

$$3(-35) + y - 22 = 13$$
$$-105 + y - 22 = 13$$
$$y - 127 = 13$$
$$y = 140$$

The solution is $x = -35$, $y = 140$, $z = 22$.

12. Gretchen Schmidt plans to buy shares of two stocks. One costs \$32 per share and pays dividends of \$1.20 per share. The other costs \$23 per share and pays dividends of \$1.40 per share. She has \$10,100 to spend and wants to earn dividends of \$540. How many shares of each stock should she buy?

    Let $x =$ the number of shares of the first stock
    $y =$ the number of shares of the second stock

$$32x + 23y = 10,100$$
$$1.2x + 1.4y = 540$$

Multiply equation 1 by 1.2 and equation 2 by -32 and add the results:

$$38.4x + 27.6y = 12,120$$
$$\underline{-38.4x - 44.8y = -17,280}$$
$$-17.2y = -5160$$
$$y = 300$$

Plug this into equation 2:

$$1.2x + 1.4y = 540$$
$$1.2x + 1.4(300) = 540$$
$$1.2x + 420 = 540$$
$$1.2x = 120$$
$$x = 100$$

She should buy 100 shares of the first stock and 300 shares of the second stock.

13. Joyce Pluth has money in two investment funds. Last year the first fund paid a dividend of 8% and the second dividend of 2% and Joyce received a total of \$780. This year the first fund paid a 10% dividend and the second only 1% and Joyce received \$810. How much does she have invested in each fund?

    Let $x =$ the amount invested in the first fund
    $y =$ the amount invested in the second fund

$$.08x + .02y = 780$$

$$.1x + .01y = 810$$

Multiply the second equation by -2 and add to the first equation:

$$\begin{aligned} .08x + .02y &= 780 \\ \underline{-.2x - .02y} &\underline{= -1620} \\ -.12x &= -840 \\ x &= 7000 \end{aligned}$$

Substitute this into the first equation:

$$.08x + .02y = 780$$

$$\begin{aligned} .08(7000) + .02y &= 780 \\ 560 + .02y &= 780 \\ .02y &= 220 \\ y &= 11,000 \end{aligned}$$

Joyce has invested \$7000 in the first fund and \$11,000 in the second fund.

14. Shirley Cicero has \$16,000 invested in Boeing and GE stock. The Boeing stock currently sells for \$30 per share and the GE stock for \$70 per share. If GE stock triples its value and the Boeing stock goes up 50%, her stock will be worth \$34,5000. How many shares of each stock does she own?

    Let x = number of shares of Boeing stock.

    Let y = number of shares of GE stock.

    $$30x + 70y = 16,000$$

    If the price of the GE stock triples then it will be worth $70 \cdot 3 = \$210$ per share. If the price of the Boeing stock increases by 50%, it will be worth $30 \cdot 1.5 = \$45$ per share. Therefore we also get the following equation:

    $$45x + 210y = 34500$$

    Multiply the first equation by -3:

    $$-90x - 210y = -48,000$$

    $$\begin{aligned} \underline{45x + 210y} &\underline{= 34500} \\ -45x &= -13,500 \\ x &= 300 \end{aligned}$$

    Plugging back in, we get:

$$45(300) + 210y = 34500$$
$$13,500 + 210y = 34500$$
$$210y = 21000$$
$$y = 100$$

Therefore she has 300 shares of Boeing stock and 100 shares of GE stock.

15. Pretzels cost \$3 per pound, dried fruit \$4 per pound and nuts \$8 per pound. how many pounds of each should be used to produce 140 pounds of trail mix costed \$6 per pound in which threa re twice as many pretzels (by weight) as dried fruit?

    Let p = number of pounds of pretzels

    f = number of pounds of dried fruit

    n = number of pounds of nuts

$$p = 2f \Rightarrow 2f - p = 0$$
$$p + f + n = 140$$
$$3p + 4f + 8n = 140(6)$$

Using the first equation to rewrite the other two:

$$p = 2f$$
$$2f + f + n = 140 \Rightarrow 3f + n = 140$$
$$3(2f) + 4f + 8n = 840 \Rightarrow 10f + 8n = 840$$

Solving the second equation for n, we get

$$n = 140 - 3f$$

and so the last equation becomes

$$10f + 8(140 - 3f) = 840$$
$$10f + 1120 - 24f = 840$$
$$-14f = -280$$
$$f = 20$$

Substituting back:

$$n = 140 - 3f = 140 - 3(20) = 140 - 60 = 80$$
$$p = 2f = 2(20) = 40$$

Therefore, they should use 40 pounds of pretzels, 20 pounds of dried fruit and 80 pounds of nuts.

16. You are given \$144 in one, five, and ten dollar bills. There are two more ten dollar bills than five dollar bills. How many bills of each type are there?

Let $x =$ number of one dollar bills

$y =$ number of five dollar bills

$z =$ number of ten dollar bills

We get the equations:

$$x + y + z = 35$$
$$x + 5y + 10z = 144$$
$$z = y + 2 \Rightarrow z - y = 2$$

We can use the third equation to rewrite the first two:

$$x + y + y + 2 = 35 \Rightarrow x + 2y = 33$$
$$x + 5y + 10(y + 2) = 144 \Rightarrow x + 5y + 10y + 20 = 144 \Rightarrow x + 15y = 124$$
$$x = y + 2$$

Solve the first equation for x: $x = 33 - 2y$ and substitute into the second equation:

$$33 - 2y + 15y = 91$$
$$13y = 91$$
$$y = 7$$

$$x = 33 - 2y = 33 - 2(7) = 19$$

$$z = y + 2 = 7 + 2 = 9$$

therefore there are 19 ones, 7 fives, and 9 tens.

17. Use matrices and your calculator to solve the following systems;

(a) $4x - y - 2z = 4$
$x - y - \dfrac{1}{2}z = 1$
$2x - y - z = 8$

$$\left(\begin{array}{ccc|c} 4 & -1 & -2 & 4 \\ 1 & -1 & -\frac{1}{2} & 1 \\ 2 & -1 & -1 & 8 \end{array}\right)$$

By calculator:

$$\left(\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

Because this last row says that $0 = 1$ the system is inconsistent and it has no solutions.

(b) $x - z = -3$
$y + z = 6$
$2x - 3z = -9$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & -3 \\ 0 & 1 & 1 & 6 \\ 2 & 0 & -3 & -9 \end{array}\right)$$

By calculator:

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

The solution is $(0, 3, 3)$, i.e. $x = 0$, $y = 3$, and $z = 3$

(c) $x - 2y + 3z = 4$
$2x + y - 4z = 3$
$-3x + 4y - z = -2$

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 2 & 1 & -4 & 3 \\ -3 & 4 & -1 & -2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

The solution is $x = 4$, $y = 3$, $z = 2$.