Math 199
Fall, 2004
Review for Exam 1

Here are some things to look at as you prepare for the exam. Remember to look at quizzes and homework problems also. The actual exam will consist of a large number of true/false and multiple choice problems, but the problems below will help you understand the concepts covered on the exam. Try to do as many of these as you can without looking in your notes or book for guidance.

The first exam will be on Tuesday, 28 September and will cover material in Chapters 1 and §2.1.

1. Solve for x: $4 - 5x = 9$

$$4 - 5x = 9$$
$$-5x = 5$$
$$x = -1$$

2. Solve for x: $(x-3)(x-2) = 0$

$$x - 3 = 0 \text{ or } x - 2 = 0$$
$$x = 3 \text{ or } x = 2$$

3. Solve for x: $8x^2 = 8x - 3$

$$8x^2 - 8x + 3 = 0$$

This cannot be factored, so use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{8 \pm \sqrt{64 - 498) * 3)}}{2(8)}$$
$$= \frac{8 \pm \sqrt{64 - 96}}{16}$$

Since $64 - 96 = -32$ is less than zero, this equation has no real solutions.

4. Solve for b: $(b+7)^2 = 5$

$$b^2 + 14b + 49 = 5$$
$$b^2 + 14b + 44 = 0$$
$$b = \frac{-14 \pm \sqrt{14^2 - 4(1)(44)}}{2}$$

$$= \frac{-14 \pm \sqrt{196 - 176}}{2}$$
$$= \frac{-14 \pm \sqrt{20}}{2}$$
$$= \frac{-14 \pm 2\sqrt{5}}{2}$$
$$= \frac{2(-7 \pm \sqrt{5}}{2}$$
$$= -7 \pm \sqrt{5}$$

5. Solve for k: $9k^2 + 6k = 2$

$$9k^2 + 6k - 2 = 0$$
$$k = \frac{-6 \pm \sqrt{6^2 - 4(9)(-2)}}{2 \cdot 9}$$
$$= \frac{-6 \pm \sqrt{36 + 72}}{18}$$
$$= \frac{-6 \pm \sqrt{108}}{18}$$
$$= \frac{-6 \pm 6\sqrt{3}}{18}$$
$$= \frac{6(-1 \pm \sqrt{3}}{18}$$
$$= \frac{-1 \pm \sqrt{3}}{3}$$

6. Find the $x-$intercept(s) and $y-$intercept of $x - 2y = 3$

   The $x-$intercepts occur when $y = 0$:

$$x - 2(0) = 3$$
$$x = 3$$

   The $y-$intercept occurs when $x = 0$:

$$0 - 2y = 3$$
$$-2y = 3$$
$$y = -\frac{3}{2}$$

7. Find the $x-$intercept(s) and $y-$intercept of $y = x^2 - 9$
   The $x-$intercepts occur when $y = 0$:

$$0 = x^2 - 9 = (x+3)(x-3)$$
$$x + 3 = 0 \text{ or } x - 3 = 0$$
$$x = -3 \text{ or } x = 3$$

   The $y-$intercept occurs when $x = 0$:

$$y = 0^2 - 9$$
$$y = -9$$

8. For each of the following, find the equation of a line satisfying the specified conditions:

   (a) Through $(-1, 4)$ and $(2, 3)$

$$m = \frac{\text{rise}}{\text{run}} = \frac{4-3}{2-(-1)} = \frac{1}{3}$$

$$y - 4 = \frac{1}{3}(x - (-1))$$
$$y - 4 = \frac{1}{3}(x + 1)$$

   (b) Through $(5, -3)$ and perpendicular to $x = 3y$
   The slope of $x = 3y$ is $y = \frac{1}{3}$. The line perpendicular to that will have slope -3. So:

$$y - (-3) = -3(x - 5)$$
$$y + 3 = -3(x - 5)$$

   (c) Through $(7, 11)$ and parallel to $3x + 8y = 0$
   First we need to find the slope of $3x + 8y = 0$:

$$8y = -3x$$
$$y = -\frac{3}{8}x$$

   So it's slope if $-\frac{3}{8}$. A line parallel to this will have the same slope, so it's equation is:

$$y - 11 = -\frac{3}{8}(x - 7)$$

   (d) with $x-$intercept -3 and $y-$intercept 5
   This means that the points $(-3, 0)$ and $(0, 5)$ are on the line. Therefore the line has slope:

$$m = \frac{5-0}{0-(-3)} = \frac{5}{3}$$

And the equation is:

$$y - 5 = \frac{5}{3}(x - 0)$$
$$y - 5 = \frac{5}{3}x$$

9. Chris and Josh have received walkie-talkies for Christmas. If they leave from the same point at the same time, Chris walking north at 2.5 mph and Josh walking east at 3 mph, how long will they be able to talk to each other if the range of the walkie-talkies is 4 miles? Round your answer to the nearest minute.

   Let
   $x =$ the number of hours that Chris and Josh walk
   $2.5x =$ the distance that Chris walks
   $3x =$ the distance that Josh walks

   Then we can use the Pythagorean theorem since the other side is the 4 mile distance between Chris and Josh:

$$(3x)^2 + (2.5x)^2 = 4^2$$
$$9x^2 + 6.25x^2 = 16$$
$$15.25x^2 = 16$$
$$x^2 = \frac{16}{15.25}$$
$$x = \pm\sqrt{\frac{16}{15.25}}$$
$$x \approx \pm 1.02$$

   We can ignore the negative solution since it would indicate negative time. Therefore, since we want the answer in minutes, the answer is $1.02 \cdot 60 \approx 61$ minutes before the boys are out of range.

10. A plane flies nonstop from New York to London, cities which are about 3500 miles apart. After one hour and 6 minutes in the air, the plane passes over Halifax, Nova Scotia, which is 600 miles from New York. Estimate the flying time from New York to London.

    The plane flies 66 minutes to Halifax. Therefore the time that it takes to 600 miles is proportional to the time it takes to fly the 3500 miles for the entire trip:

$$\frac{66}{600} = \frac{x}{3500}$$
$$3500(66) = 600x$$
$$231,000 = 600x$$
$$x = 385$$

    This is the number of minutes it takes. To find the number of hours, divide by 60, and the trip takes about 6.42 hours.

11. On vacation, Le Hong averaged 50 mph traveling from Denver to Minneapolis. Returning by a different route that covered the same number of miles , he averaged 55 mph. What is the distance between the two cites if his total traveling time was 32 hours?

    Let $x =$ the number of hours at 50 mph. Then $32 - x =$the number of hours at 55 mph. The distance travelled at 50 mph is $50x$ and the distance travelled at 55 mph is $55(32 - x)$. Since the distance each way is equal, we have that:

$$50x = 55932 - x)$$
$$50x = 1760 - 55x$$
$$105x = 1760$$
$$x \approx 16.76$$

    Therefore, $50x \approx 50(16.76) = 838$ miles between the two cities.

12. Joan wants to buy a rug for a room that is 12 feet by 15 feet. She wants to leave a uniform strip of floor around the rug. She can afford 108 square feet of carpeting. What dimensions should the rug have?

    Let x be the amount of floor on each side that is not covered by the rug. We know that the dimensions of the rug will be $12 - 2x$ by $15 - 2x$ and that the area of the rug is 108 square feet. Therefore:

$$(12 - 2x)(15 - 2x) = 108$$
$$180 - 30x - 24x + 4x^2 = 108$$
$$72 - 54x + 4x^2 = 0$$
$$2x^2 - 27x + 36 = 0$$
$$(x - 12)(2x - 3) = 0$$
$$x = 12 \text{ or } x = \frac{3}{2}$$

    If $x = 12$, then there is no rug, so the amount left uncovered by the rug on each side is 1.5 feet. Therefore the dimensions of the rug are

$$\begin{array}{ccc} 12 - 2x & \text{by} & 15 - 2x \\ 12 - 2\frac{3}{2} & \text{by} & 15 - 2\frac{3}{2} \\ 12 - 3 & \text{by} & 15 - 3 \\ 9 & \text{by} & 12 \end{array}$$

13. Deer ticks cause concern because the can carry Lyme disease. One study found a relationship between the density of acorns produced in the fall and the density of deer tick larvae the following spring. the relationship can be approximated by the linear equation

$$y = 34x + 230$$

    where x is the number of acorns per square meter in the fall and y is the number of deer tick larvae per 400 square meters the following spring. According to this formula, approximately how many acorns per square meter would result in 1000 deer ticks larvae per 400 square meters?

We want to solve:

$$1000 = 34x + 230$$
$$770 = 34x$$
$$x \approx 22.64$$

We would need to see approximately 23 acorns per square meter to expect 1000 deer ticks per 400 square meters.

14. One car rental firm charges \$75 for a weekend rental (Friday afternoon through Monday morning) with unlimited mileage. A second firm charges \$50 plus 5 cents per mile. For what range of miles driven is the second firm cheaper?

    Let $x =$ the number of miles driven. Then the inequality is

$$75 > 50 + .05x$$
$$25 > .05x$$
$$500 > x$$

    It is cheaper at the second firm if you drive less than 500 miles.

15. State the definition of function.

    A function is a rule that assigns to each input a unique output.

16. Which of the following define a function? Explain.

    (a) $y = \sqrt{x}$
    Recall that a function is a rule that assigns to each input a unique output. This is a function because each value of x determines one and only one value of y.

    (b) $x = y^2 + 1$
    This rule does not define y as a function of x. A given value of x may define two values for y. For example, if $x = 10$, then $y = 3$ or $y = -3$.

    (c) $x = \mid y \mid$
    This rule does not define y as a function of x. A given value of x may define two values for y. For example, if $x = 4$, then $y = 4$ or $y = -4$

    (d) $y = \mid x \mid$
    This is a function because each value of x determines one and only one value of y.

17. For each of the following functions, find

    $f(6)$ $\quad f(-2)$ $\quad f(p)$ $\quad f(r+1)$ $\quad f(k-5)$ $\quad f(x+h)$

    (a) $f(x) = 4x - 1$

$$\begin{aligned} f(6) &= 4(6) - 1 = 24 - 1 = 23 \\ f(-2) &= 4(-2) - 1 = -8 - 1 = -9 \\ f(p) &= 4p - 1 \\ f(r+1) &= 4(r+1) - 1 = 4r + 4 - 4 = 4r + 3 \\ f(k-5) &= 4(k-5) - 1 = 4k - 20 - 1 = 4k - 21 \\ f(x+h) &= 4(x+h) - 1 = 4x + 4h - 1 \end{aligned}$$

(b) $f(x) = -x^2 + 2x - 4$

$$\begin{aligned}
f(6) &= -(6^2) + 2(6) - 4 \\
&= -36 + 12 - 4 = -28 \\
f(-2) &= -(-2)^2 + 2(-2) - 4 \\
&= -4 - 4 - 4 = -12 \\
f(p) &= -p^2 + 2p - 4 \\
f(r+1) &= -(r+1)^2 + 2(r+1) - 4 \\
&= -(r^2 + 2r + 1) + 2r + 2 - 4 \\
&= -r^2 - 3 \\
f(k-5) &= -(k-5)^2 + 2(k-5) - 4 \\
&= -(k^2 - 10k + 25) + 2k - 10 - 4 \\
&= -k^2 + 12k - 39 \\
f(x+h) &= -(x+h)^2 + 2(x+h) - 4
\end{aligned}$$

(c) $f(x) = 8 - x - x^2$

$$\begin{aligned}
f(6) &= 8 - 6 - 6^2 = 2 - 36 = -34 \\
f(-2) &= 8 - (-2) - (-2)^2 = 8 + 2 - 4 = 10 - 4 = 6 \\
f(p) &= 8 - p - p^2 \\
f(r+1) &= 8 - (r+1) - (r+1)^2 \\
&= 8 - r - 1 - (r^2 + 2r + 1) \\
&= 7 - r - r^2 - 2r - 1 \\
&= 6 - 3r - r^2 \\
f(k-5) &= 8 - (k-5) - (k-5)^2 \\
&= 8 - k + 5 - (k^2 - 10k + 25) \\
&= 13 - k - k^2 + 10k - 25 \\
&= -12 + 9k - k^2 \\
f(x+h) &= 8 - (x+h) - (x+h)^2
\end{aligned}$$

(d) $\dfrac{x^2+2}{x-6}$

$$\begin{aligned}
f(6) &= \text{undefined since we cannot divide by 0} \\
f(-2) &= \frac{(-2)^2 + 2}{(-2) - 6} = \frac{4+2}{-8} = \frac{6}{-8} = -\frac{3}{4} \\
f(p) &= \frac{p^2+2}{p-6} \\
f(r+1) &= \frac{(r+1)^2 + 2}{(r+1) - 6} = \frac{r^2 + 2r + 1 + 2}{r + 1 - 6} \\
&= \frac{r^2 + 2r + 3}{r - 5} \\
f(k-5) &= \frac{(k-5)^2 + 2}{(k-5) - 6} \\
&= \frac{k^2 - 10k + 25 + 2}{k - 5 - 6} \\
&= \frac{k^2 - 10k + 27}{k - 11} \\
f(x+h) &= \frac{(x+h)^2 + 2}{(x+h) - 6}
\end{aligned}$$

18. Let $f(x) = x^2 + x + 1$. Find each of the following:

(a) $f(3) = 3^2 + 3 + 1 = 9 + 3 + 1 = 13$

(b) $f(1) = 1^2 + 1 + 1 = 3$

(c) $f(4) = 4^2 + 4 + 1 = 16 + 4 + 1 = 21$

(d) Based on your answers above, is it true that $f(a + b) = f(a) + f(b)$ for all real numbers a and b?

No because

$$f(3) + f(1) = 13 + 3 = 16$$

but $f(3 + 1) = f(4) = 21$

19. Graph each of the following functions:

(a) $f(x) = \mid x \mid -3$

This is the absolute value function, shifted down 3.

(b) $f(x) = [x - 3]$

Oops - skip this - we haven't defined this notation - pretend it is the absolute value function shifted right three units.

(c) $f(x) = \begin{cases} -4x + 2 & \text{if } x \leq 1 \\ 3x - 5 & \text{if } x > 1 \end{cases}$

This is the line with $y-$intercept 2 and slope -4 for all values of x smaller than or equal to one. It is the line with $y-$intercept 5 and slope 3 for all values of x bigger than one.

(d) $f(x) = \begin{cases} \mid x \mid & \text{if } x < 3 \\ 6 - x & \text{if } x \geq 3 \end{cases}$

This is the absolute value function for all values of x less than three. It is the line with $y-$intercept 6 and slope -1 for all values of x bigger than or equal to 3.

20. Let f be a function that gives the cost to rent a floor polisher for x hours. The cost is a flat \$3 for renting the polisher plus \$4 per day or fraction of day for using the polisher.

(a) Find the cost function for renting the polisher.

This is harder than I meant, and you should graph it instead.

(b) David Fleming wants to rent a polisher, but he can spend no more than \$15. At most how many days can he use it?

If he can only spend \$15 dollars, he can use the polisher for at most 3 days because:

Time	**Cost**
to use the polisher at all	\$3
1 day	\$7
2 days	\$11
3 days	\$15
4 days	\$19

21. If it costs \$300 to produce 8 units, and the fixed costs are \$60, find:

(a) the linear cost function

$$C(x) = mx + b$$
$$b = 60 \text{and} C(8) = 300$$
$$C(x) = mx + 60$$
$$300 = m(8) + 60$$
$$8m = 240$$
$$m = 30$$
$$C(x) = 30x + 60$$

(b) the marginal cost
this is the slope of the linear cost function, so it is \$30

(c) the average cost per unit to produce 100 units

$$C(100) = 30(100) + 60 = 3000 + 60 = 3060$$
$$\text{average cost for 100 units} = \frac{3060}{100} = \$30.6$$

22. If the fixed costs are \$2000, and 36 units cost \$8480 to make, find:

(a) the linear cost function

$$C(x) = mx + 2000$$
$$C(36) = 8480 = 36m + 2000$$
$$6480 = 36m$$
$$m = 180$$
$$C(x) = 180x + 2000$$

(b) the marginal cost
This is the slope of the linear cost function, so it is \$180

(c) the average cost per unit to produce 100 units

$$C(100) = 180(100) + 2000 = 18000 + 2000 = 20,000$$
$$\text{average cost for 100 units} = \frac{20,000}{100} = \$200$$

23. If it costs \$445 to make twelve units, and \$1585 to make 50 units, find:

(a) the linear cost function

$$C(x) = mx + b$$

The points $(12, 445)$ and $(50, 1585)$ are on the line, so

$$m = \frac{1585 - 445}{50 - 12} = \frac{1140}{38} = 30$$
$$C(x) = 30x + b$$
$$445 = 30(12) + b$$
$$445 = 360 + b$$
$$85 = b$$
$$C(x) = 30x + 85$$

(b) the marginal cost

This is the slope of the linear cost function, so it is \$30

(c) the average cost per unit to produce 100 units

$$C(100) = 30(100) + 85 = 3000 + 85 = 3085$$
$$\text{average cost for 100 units} = \frac{3085}{100} = \$30.85$$

24. The cost of producing x ink cartridges for a printer is given by $C(x) = 24x + 18,000$. Each cartridge can be sold for \$28.

(a) What are the fixed costs?

This is the $y-$intercept of the linear cost function, so it is $\$18,000$

(b) Find the revenue function.

$$R(x) = (\text{number sold})(\text{price per unit})$$
$$R(x) = 28x$$

(c) Find the profit function.

$$P(x) = R(x) - C(x)$$
$$P(x) = 28x - (24x + 18,000)$$
$$P(x) = 28x - 24x - 18000$$
$$P(x) = 4x - 18,000$$

(d) Find the break-even point.

This is where

$$C(x) = 0$$
$$4x - 18,000 = 0$$
$$4x = 18000$$
$$x = 4500 \text{ cartridges}$$

(e) If the company sells exactly the number of cartridges needed to break-even, what is its revenue?

$$R(x) = 28x$$
$$R(4500) = 28(4500) = \$126,000$$

25. Suppose the demand and price for the HBO cable channel are related by $p = -0.5q + 30.95$, where p is the monthly price in dollars, and q is measured in millions of subscribers. If the price and supply are related by $p = 0.3q + 2.15$, what are the equilibrium quantity and price?

$$-0.5q + 30.95 = 0.3q + 2.15$$
$$-.8q = -28.8$$
$$q = 36\text{million subscribers}$$

$$-.5(36) + 30.95 = \$12.95\text{per month}$$

The equilibrium quantity is 36 million subscribers and the equilibrium price is \$12.95 per month.

26. For the following functions, answer the following questions:

- Does the parabola open up or down?
- What is the vertex?
- What is the axis of symmetry?
- What are the $x-$intercepts?
- What is the $y-$intercept?

(a) $f(x) = x^2 + 6x - 2$

- Does the parabola open up or down?
  up because the leading coefficient is positive
- What is the vertex?

$$f(x) = x^2 + 6x - 2 = (x^2 + 6x + 9 - 9) - 2 = (x + 3)^2 - 11$$

  The vertex is $(-3, -11)$
- What is the axis of symmetry?

$$x = -2$$

- What are the $x-$intercepts?

$$0 = x^2 + 6x - 2$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 - 4(1)(-2)}}{2}$$
$$= \frac{-6 \pm \sqrt{36 + 8}}{2} = \frac{-6 \pm \sqrt{44}}{2}$$

$$= \frac{-6 \pm 2\sqrt{11}}{2} = -3 \pm \sqrt{11}$$

the $x-$intercepts are $(-3+\sqrt{11}, 0)$ and $(-3-\sqrt{11}, 0)$

- What is the $y-$intercept?
  Plug in $x = 0$:

$$y = 0^2 + 6(0) - 2 = -2$$

  The $y-$intercept is $(-2, 0)$

(b) $f(x) = -4x^2 + 8x + 3$

- Does the parabola open up or down?
  Opens down because the leading coefficient is negative.
- What is the vertex?

$$\begin{aligned} f(x) = -4x^2 + 8x + 3 &= -4(x^2 - 2x) + 3 \\ &= -4(x^2 - 2x + 1 - 1) + 3 \\ &= -4\left((x-1)^2 - 1\right) + 3 \\ &= -4(x-1)^2 + 4 + 3 \\ &= -4(x-1)^2 + 7 \end{aligned}$$

  the vertex is at $(1, 7)$

- What is the axis of symmetry?

$$x = 1$$

- What are the $x-$intercepts?

$$\begin{aligned} 0 &= -4x^2 + 8x + 3 \\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} &= \frac{-8 \pm \sqrt{64 - 4(-4)(3)}}{2(-4)} \\ &= \frac{-8 \pm \sqrt{64 + 48}}{-8} \\ &= \frac{-8 \pm \sqrt{112}}{-8} \\ &= \frac{-8 \pm 4\sqrt{7}}{-8} \\ &= 1 \pm \frac{\sqrt{7}}{-2} \end{aligned}$$

  the $x-$intercepts are $(1 - \frac{\sqrt{7}}{2}, 0)$ and $(1 + \frac{\sqrt{7}}{2}, 0)$

- What is the $y-$intercept?
  Plug in $x = 0$:

$$f(0) = -4(0)^2 + 8(0) + 3 = 3$$

27. Suppose an investor kept track of the profit, P, she made on her portfolio. At time t months after she began investing, $P(t) = -4t^2 + 32t - 20$. At what time is her profit largest?

$$\begin{aligned} P(t) &= -4t^2 + 32t - 20 \\ &= -4(t^2 - 8t) - 20 \\ &= -4(t^2 - 8t + 16 - 16) - 20 \\ &= -4\left((t-4)^2 - 16\right) - 20 \\ &= -4(t-4)^2 + 64 - 20 \\ &= -4(t-4)^2 + 44 \end{aligned}$$

Therefore her profit is the largest when $t = 4$, so at 4 months.

28. The height h (in feet) of a rocket at t seconds after liftoff is given by $h = -16t^2 + 800t$.

(a) How long does it take the rocket to reach 300 feet?
Let $h = 300$, and solve for t:

$$\begin{aligned} 300 &= -16t^2 + 800t \\ 16t^2 - 800t + 300 &= 0 \\ t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} &= \frac{800 \pm \sqrt{(-800)^2 - 4(16)(300)}}{2(16)} \\ &= \frac{800 \pm \sqrt{640,000 - 19,200}}{32} \\ &= \frac{800 \pm \sqrt{620,800}}{32} \\ &= \frac{800 \pm 80\sqrt{97}}{32} \\ &= 25 \pm 2.5\sqrt{97} \end{aligned}$$

It first reaches 300 feet (on the way up) at time $t = 25 - 2.5\sqrt{97} \approx .377$sec

(b) What is the maximum height of the rocket?
This happens at the vertex:

$$\begin{aligned} h &= -16t^2 + 800t \\ &= -16(t^2 - 50t) \\ &= -16(t^2 - 50t + 625 - 625) \\ &= -16\left((t-25)^2 - 625\right) \\ &= -16(t-25)^2 + 10,000 \end{aligned}$$

The maximum height is the $y-$coordinate of the vertex, so the maximum height is 10,000 feet.

29. Graph the following polynomial functions:

    (a) $f(x) = x^3 - x$
    This has the end behavior of an odd degree polynomial with positive leading coefficient, and has $x-$intercepts at $0, 1, -1$.

    (b) $f(x) = x(x-2)(x+3)$
    This has the end behavior of an odd degree polynomial with positive leading coefficient, and has $x-$intercepts at $0, 2, -3$

    (c) $f(x) = x^4 - 7x^2 - 8$
    This has the end behavior of an even degree polynomial with positive leading coefficient, and has $x-$intercepts at $\sqrt{8}, -\sqrt{8}$.

30. For the following functions, do the following:

    - Find the vertical asymptotes.
    - Find the horizontal asymptote
    - Find the $x-$intercepts
    - Graph the function

    (a) $f(x) = \dfrac{1}{x-3}$
    - Find the vertical asymptotes.
      These happen where the denominator is 0, so at $x = 3$
    - Find the horizontal asymptote
      Since the degree of the denominator is larger than the degree of the numerator, the horizontal asymptote is $y = 0$
    - Find the $x-$intercepts
      These happen where the numerator is 0, which never happens for this function.

    (b) $g(x) = \dfrac{5x-2}{4x^2-4x+3}$
    - Find the vertical asymptotes.
      These happen where the denominator is zero, so where

$$4x^2 - 4x + 3 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 - 4(4)(3)}}{2(4)}$$

$$= \frac{4 \pm \sqrt{16 - 48}}{8}$$

      Since we cannot take the square root of a negative number, there are no vertical asymptotes for this function.
    - Find the horizontal asymptote
      Since the degree of the denominator is larger than the degree of the numerator, the horizontal asymptote is $y = 0$

- Find the $x-$intercepts
  These happen where the numerator is 0, so

$$5x - 2 = 0$$

$$5x = 2$$

$$x = \frac{2}{5}$$

(c) $h(x) = \dfrac{x^2 - 4}{x + 2}$
First let's factor the numerator and denominator of the function:

$$h(x) = \frac{x^2 - 4}{x + 2} = \frac{(x - 2)(x + 2)}{x + 2}$$

- Find the vertical asymptotes.
  These happen where the denominator is zero, as long as these problems don't cancel with the numerator, so we check where

$$x + 2 = 0$$

$$x = -2$$

  However, this problem cancels with the numerator, and so $x = -2$ is a place where we have a hole in the graph, and there are no vertical asymptotes
  In fact, this function looks just like the function $f(x) = x - 2$ except for the hole at $x = -2$
- Find the horizontal asymptote
  There is no horizontal asymptote, since this function looks like a line.
- Find the $x-$intercepts
  This is where the numerator is zero, so where

$$x - 2 = 0$$

$$x = 2$$

- Graph the function
  the graph looks like a line with $y-$intercept -2 and slope 1

31. A cost-benefit curve for pollution control is given by

$$y = \frac{9.2x}{106 - x}$$

where y is the cost in thousands of dollars of removing x percent of a specific industrial pollutant.

(a) Find y if $x = 50$
if $x = 50$ then

$$y = \frac{9.2(50)}{106 - 50} = 8.2 \text{ thousand dollars} = \$8200$$

(b) Find y if $x = 98$

If $x = 98$, then

$$y = \frac{9.2(98)}{106 - 98} = 112.7 \text{ thousand dollars} = \$112,700$$

(c) What percent of the pollutant can be removed for $22,000?

This means $y = \$22$, and so we have:

$$22 = \frac{9.2x}{106 - x}$$

$$22(106 - x) = 9.2x$$

$$2332 - 22x = 9.2x$$

$$2332 = 31.2x$$

$$x \approx 74.7$$

So about 75% of pollutants can be removed for $22,000.