
Solve all problems. Show all work. NO WORK = NO CREDIT. Leave all answers in exact values, except on word problems. If you need to find the vertex of a parabola, show all steps necessary for completing the square.

1. (3 points) State the definition of function.
2. (2 points) Find the slope of the line $x = 5$.
The slope of a vertical line is undefined.
3. (6 points) Find the equation of the line through $(-2, 3)$ which is perpendicular to the line $3x + 4y = 8$.

$$3x + 4y = 8$$

$$4y = 8 - 3x$$

$$y = 2 - \frac{3}{4}x$$

$$m = -\frac{3}{4}$$

Therefore the slope we want is $\frac{4}{3}$:

$$y - 3 = \frac{4}{3}(x + 2)$$

$$y = \frac{4}{3}x + \frac{8}{3} + \frac{9}{3} = \frac{4}{3}x + \frac{17}{3}$$

4. Look at the quadratic function $f(x) = -6(x + 3)^2 - 5$.
 - (a) (2 points) Which way does the parabola open?
down
 - (b) (5 points) What is the vertex of the parabola?
 $(-3, -5)$
 - (c) (2 points) What is the axis of symmetry of the parabola?
 $x = -3$
 - (d) (3 points) What is the y -intercept of the parabola?

$$f(0) = -6(0 + 3)^2 - 5 = -6(-9) - 5 = -54 - 5 = -59$$

$$(0, -59)$$

- (e) (5 points) What is/are the x -intercept(s) of the parabola?

There are no x -intercepts since the vertex is below the x -axis and it opens down.

5. Look at the rational function $g(x) = \frac{(x-3)(x-2)}{(x-2)(x+2)} = \frac{x^2 - 5x + 6}{x^2 - 4}$.

- (a) (5 points) What is the domain of the function?

Need for the bottom to not equal zero, so $x \neq -2, x \neq 2$

- (b) (3 points) What is the horizontal asymptote of the function?

$y = 1$, since the degrees of the numerator and denominator are the same.

- (c) (2 points) What is/are the vertical asymptote(s) of the function?

$$x = -2$$

- (d) (5 points) What is/are the x -intercept(s) of the function?

$$x = 3$$

- (e) (3 points) What is the y -intercept of the function?

$$\frac{0-3}{0+2} = \frac{-3}{2}$$

6. (10 points) Find the linear cost function if the marginal cost is \$150, and 100 items cost \$15,800 to produce.

$$C(x) = mx + f$$

$$C(x) = 150x + f$$

$$15800 = 150(100) + f$$

$$15800 = 15000 + f$$

$$800 = f$$

$$C(x) = 150x + 800$$

7. (10 points) The owners of a parking lot have determined that their weekly cost in dollars are given by

$$C(x) = 100x + 5000$$

where x is the number of long-term parkers. If they charge \$125 to each long-term parker, find the break even point.

$$R(x) = 125x$$

Break even when $R(x) = C(x)$:

$$100x + 5000 = 125x$$

$$5000 = 25x$$

$$200 = x$$

200 parkers

8. (8 points) Solve the following system of linear equations:

$$x + 2y + 4z = 6$$

$$x + y + z = 1$$

$$x + 3y + 5z = 10$$

Is the system inconsistent, independent or dependent?

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 6 \\ 1 & 1 & 1 & 1 \\ 1 & 3 & 5 & 10 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 3.5 \\ 0 & 0 & 1 & .5 \end{array} \right]$$

Therefore

$$x = -3$$

$$x = 3.5$$

$$x = .5$$

Therefore it is an independent system.

9. The profit (in millions of dollars) from the sale of x million units of Blue Glue is given by $P(x) = 0.7x - 25.5$. The cost is given by $C(x) = 0.9x + 25.5$.

- (a) (5 points) Find the revenue equation.

$$P = R - C$$

$$.7x - 25.5 = R - .9x - 25.5$$

$$1.6x = R(x)$$

- (b) (5 points) Find the break even point.

$$P = 0$$

$$0.7x - 25.5 = 0$$

$$0.7x = 25.5$$

$$x \approx 36.42857143$$

$$36,428,571 \text{ units}$$

10. Colleen Davis owns a factory that manufactures souvenir key chains. Her weekly profit (in hundreds of dollars) is given by

$$P(x) = -2x^2 + 60x - 120$$

where x is the number of cases of key chains sold.

- (a) (8 points) Find the largest number of cases she can sell and still make a profit.

$$\begin{aligned} P(x) &= 0 \\ -2x^2 + 60x - 120 &= 0 \\ -2(x^2 - 30x + 60) &= 0 \\ x^2 - 30x + 60 &= 0 \\ x &= \frac{30 \pm \sqrt{900 - 4(60)}}{2} \\ &= \frac{30 \pm \sqrt{900 - 240}}{2} \\ &= \frac{330 \pm \sqrt{660}}{2} \\ &\approx 27.845 \text{ or } 2.154767 \end{aligned}$$

The larger one is 27.845, so about 27 cases.

- (b) (8 points) How many cases should she sell to maximize her profit?

$$\begin{aligned} P(x) &= -2x^2 + 60x - 120 \\ &= -2(x^2 - 30x + 225 - 225) - 120 \\ &= -2(x - 15)^2 + 550 - 120 \\ &= -2(x - 15)^2 + 430 \end{aligned}$$

Therefore the vertex is $(15, 430)$, so she should sell 15 cases to maximize her profits.