Math 199
Fall, 2004
Interest Problems

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1. A sealed box contains radium. The number of grams present at time t is given by

$$Q(t) = 100e^{-0.00043t}$$

where t is measured in years. Find the amount of radium at $t = 0$ and $t = 800$.

$$Q(0) = 100e^{(-0.00043)(0)} = 100e^0 = 100 \cdot 1 = 100$$

$$Q(800) = 100e^{(-0.00043)(800)} = 100e^{-.344} \approx 70.8929$$

2. The number of internet users in the US was estimated to be 54 million in the fall of 1999 and 85 million in the fall of 2002. Find an exponential model for the data, where t = number of years after 1999, and the model has the form $f(t) = Ab^t$.

We know that $(0, 54)$ and $(3, 85)$ are on the graph. Using the first point

$$f(0) = Ab^0 = 54$$

$$A \cdot 1 = 54$$

$$A = 54$$

Therefore the equation, so far, looks like

$$f(t) = 54b^t$$

Using the second point we know

$$f(3) = 54b^3 = 85$$

$$\frac{85}{54} = b^3$$

$$\sqrt[3]{\frac{85}{54}} = b$$

$$b \approx 1.1633$$

Therefore the model is

$$f(t) = 54(1.1633)^t$$

Notice this is a model of exponential growth.

3. Boiling water at 100° C is placed in a freezer at o°C. There temperature of the water is 50° after 24 minutes. Find the temperature of the water after 96 minutes. Use Newton's Law of Cooling:

$$f(t) = T_0 + Ab^t$$

T_0 is the room temperature, so in this situation, $T_0 = 0$

Therefore we start with the model

$$f(t) = 0 + Ab^t = Ab^t$$

Doing as above, we get

$$f(t) = 100(.9715)^t$$
$$f(96) \approx 6.25°C$$

Formulas for interest calculations:

	Interest	**Future Value**	**Present Value**
Simple Interest	$I = Prt$	$A = P(1 + rt)$	$P = \dfrac{A}{1 + rt}$
Compound Interest	$I = A - P$	$A = P(1 + i)^n$	$P = \dfrac{A}{(1 + i)^n}$
Continuous Interest	$I = A - P$	$A = Pe^{rt}$	$P = \dfrac{A}{e^{rt}}$

4. Find the simple interest for an investment of \$4902 at 9.5% for 11 months.

$$I = \$426.88$$

5. Find the future value of an investment of \$3478 at 7.4% for 88 days (assume 365 days in a year). (This implies simple interest because of the short term of the loan.)

$$A = \$3540.05$$

6. Find the present value of the future amount \$459.57 if the money is invested at a simple interest rate of 8.5% for 7 months.

$$P \approx \$437.86$$

7. Find the amount of money in an account in which the initial deposit is \$2800 earning 6% interest compounded annually after 10 years.

$$A = \approx \$5014.37$$

8. Find the present value of the future amount \$42,000 if the money is deposited in an account paying 12% compounded monthly for 7 years.

$$P \approx \$18,207.65$$

9. You graduate from SHSU and get a job! You are making \$36,000 per year. You pay \$700 a month for rent, \$150 a month for insurance, and \$350 a month for your car note. You decide never to eat Ramen noodles again, so you give yourself \$200 per month for food. Cable is \$75 per month, your cell phone is \$35, and the high speed modem is another \$50. Electricity and gas cost \$250 per month.

   (a) How much "extra" money do you have per month?

   \$1190

   (b) You stick \$500 per month under your mattress. When will you have \$1 million?

   2000 months

   (c) You stick \$1000 per month under your mattress. When will you have \$1 million?

   1000 months

   (d) How much would you have to put under your mattress each month to earn \$1 million in 30 years?

   \$2777.78

10. You put \$500 into a savings account. If it earns 3% compounded annually, how much money will you have in 30 years?

    \$1213.63

11. How much money would you have to put into a savings account today, at 3% compounded annually to have \$1 million in 30 years?

    \$411,986.76

12. You put \$500 into a savings account. If it earns 3% compounded monthly, how much money will you have in 30 years?

    \$1228.42

13. How much money would you have to put into a savings account today, at 3% compounded monthly to have $1 million in 30 years?

$$\$407,026.55$$

14. You put $500 into a savings account. If it earns 3% compounded continuously, how much money will you have in 30 years?

$$\$1229.80$$

15. How much money would you have to put into a savings account today, at 3% compounded continuously to have $1 million in 30 years?

$$\$406,569.66$$

16. What is a better investment plan? How would you invest to ensure that you have $1 million in 30 years?

    Answers will vary.

17. What happens if you invest $500 per month into an account each month for the next 20 years at 3% compounded monthly. How much money will you have after 20 years?

    We'll come back to this in chapter 3.

18. If you then leave that money in an account earning 5% for the next 10 years, how much money will the account be worth when you retire?

    We'll come back to this in chapter 3.