The Incredible 9_7

My Pet Knot

Discover what you never imagined about this nine crossing knot!

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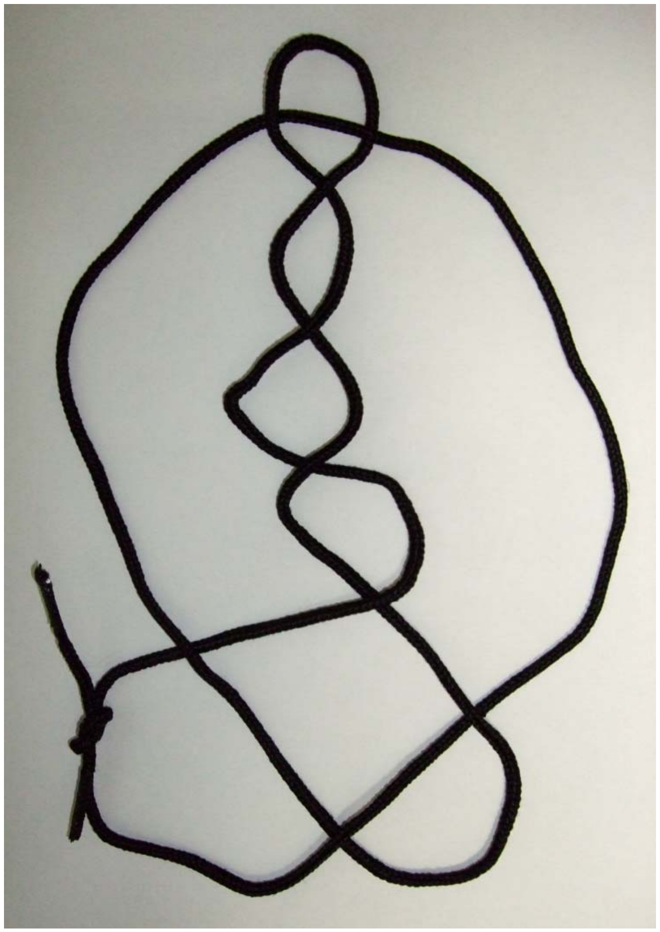
THE DIFFERENT LOOKS OF THE 9_7

- The 9_7 in three dimensions (created in Knot Plot and taken from Rolfsen's Knot Table)

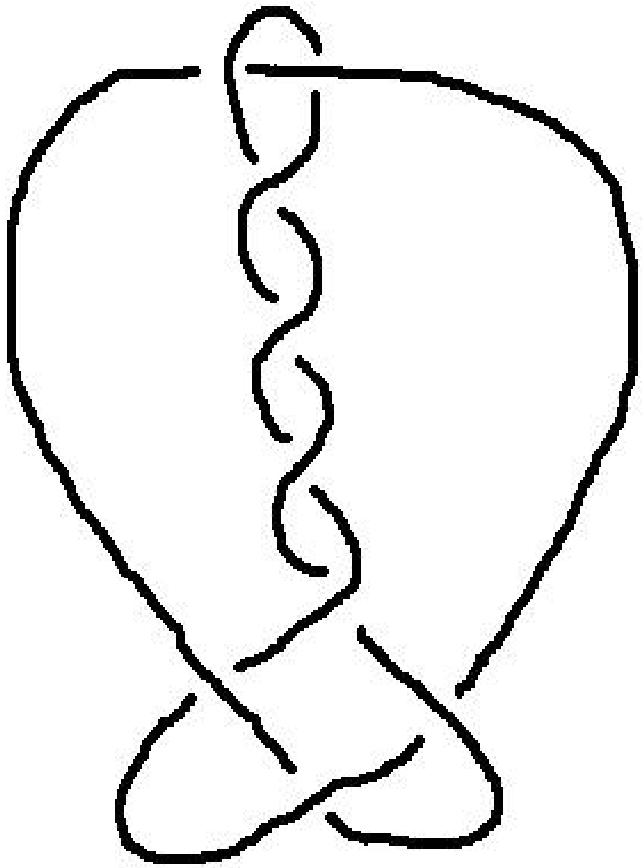
A projection of a knot is it's picture. Two knot projections represent the same knot if when you tie the knot, you could project it in a different way to get the other knot.

THE DIFFERENT LOOKS OF THE 9_7

The standard projection of the nine seven knot.



(made with a shoelace)



(drawn in paint)

Note: The nine seven is also alternating. The crossings alternate between over and under crossings. A knot is alternating if there exists a projection of that knot which is

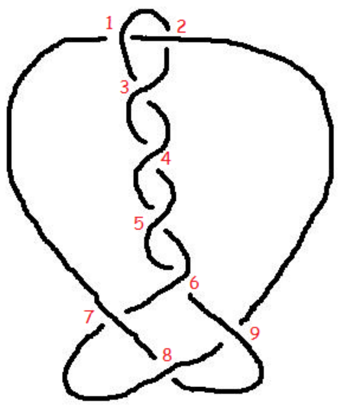
Unknotting Number

The unknotting number of a knot is the least number of crossing changes needed in ANY projection of the knot to make the unknot.

The **unknotting number** of the nine seven is

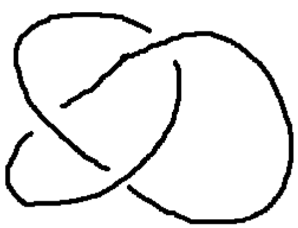
2

UNKNOTTING NUMBER CONT.



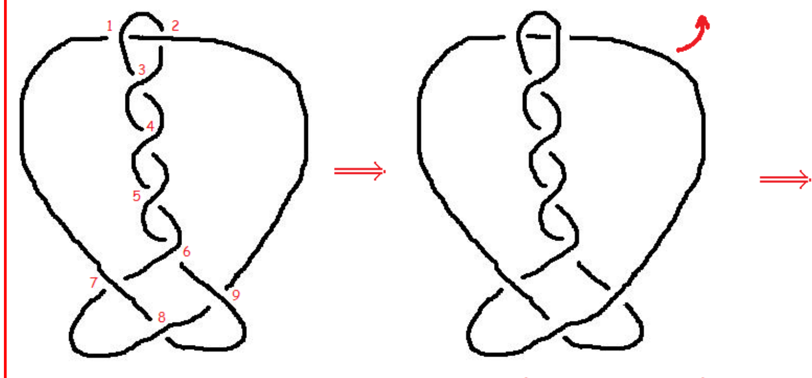
Because the nine seven has the strand that goes through the loop at the top, you always have to either change crossing # 1 or 2 so that you can untwist the center part of the knot.

When you change either crosssing # 1 or 2 and untwist the center you will end up with this projection.

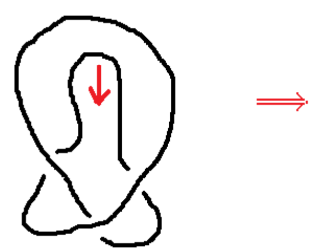


Which means that you have to change 1 or 2, AND either crossing # 7, 8, or 9.

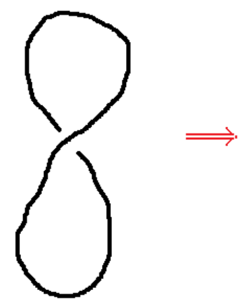
Therefore, the unknotting number is 2.



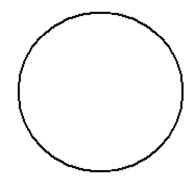
Untwist the center and pull the top strand up.



Pull this piece down.



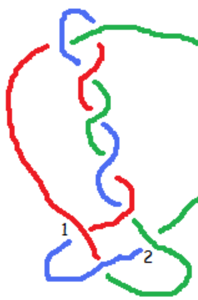
Then, untwist.



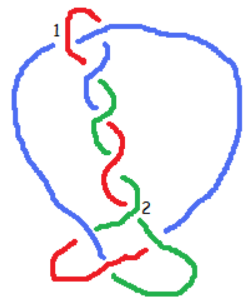
The UNKNOT

TRICOLORABILITY

I started from the top of the knot…



I started from the bottom of the knot this time…But, there are still 2 crossings that violate the 2nd rule of tricolorability.



Look at the two crossings labeled. They violate the second rule of tricolorability.

In the previous picture, if I change crossing # 1 blue and keep working downwards changing them to blue…I will get this…

This satisfies the 2nd rule but violates the 1st rule of tricolorability because I did not use three different colors.

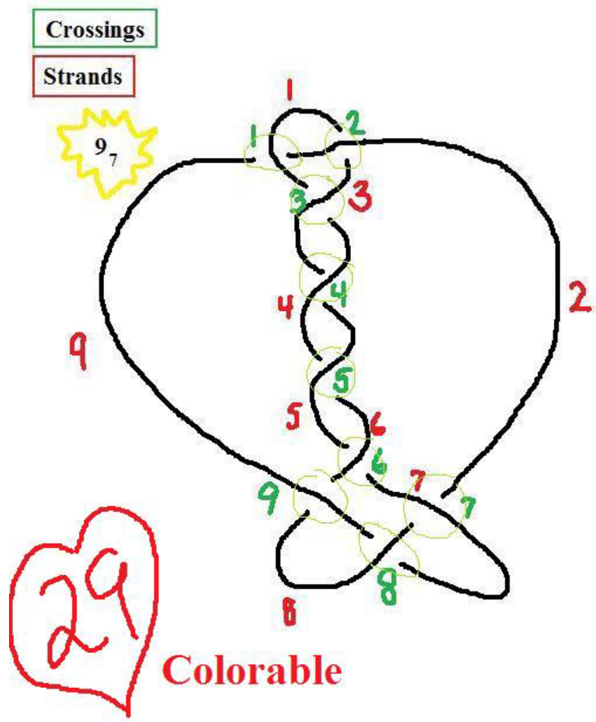
Unfortunately, the nine seven is <u>NOT</u>

T R I C O L O R A B L E

A projection of the knot is tricolorable if each of the strands in the projection can be colored one of the three different colors, so that at each crossing , either three different colors meet or all the same color come together.

P-COLORATION

My knot is not tricolorable, but it is p-colorable. P-coloration is when you label each strand with a number 0 through one less than the prime number your using to color your knot. You have to use at least two colors and at every crossing, the #'s on the two undercrossing strands must equal two times the # on the strand that crosses over.



Crossings / Strands	1	2	3	4	5	6	7	8	9
1	-2	1	1	0	0	0	0	0	
2	1	-2	0	0	0	0	1	0	
3	0	1	-2	1	0	0	0	0	
4	0	0	1	-2	1	0	0	0	
5	0	0	0	1	-2	1	0	0	
6	0	0	0	0	1	-2	0	0	1
7	0	0	0	0	0	1	-2	1	
8	0	0	0	0	0	0	1	-2	1
9	1							1	-2

Crossings are labeled 0

Understrands are labeled 1

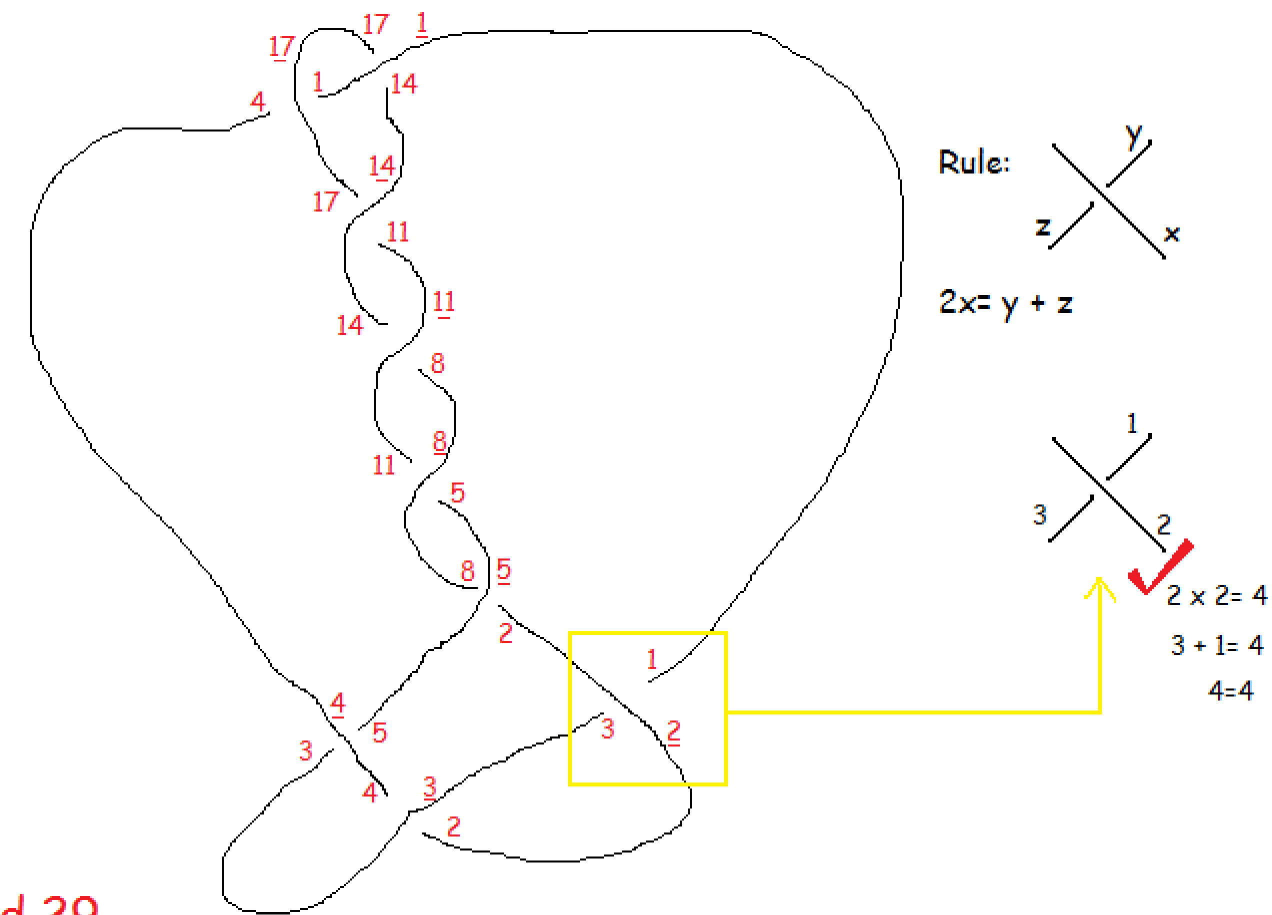
Overstrands are labeled -2

I used M-coloring to find that my knot is 29 colorable.

I labeled the crossings and strands of the nine seven to create a crossing matrix. I then plugged the matrix into a matrix calculator to discover that my knot is 29 colorable.

P-Coloration cont.

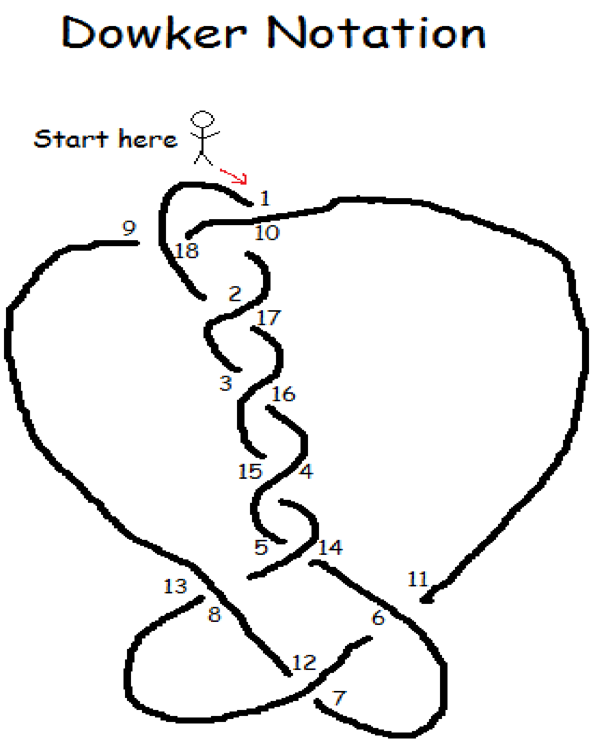
The nine seven is 29 colorable.
At every crossing 2x= y + z.



Mod 29
Label the strands 1-28

DOWKER NOTATION

To find the Dowker notation for a knot (k), pick an undercrossing at which to start. Walk around the knot, numbering crossings as you go. The Dowker notation is then gotten by listing the odd #'s used (in order) and the even #'s that correspond to those underneath.



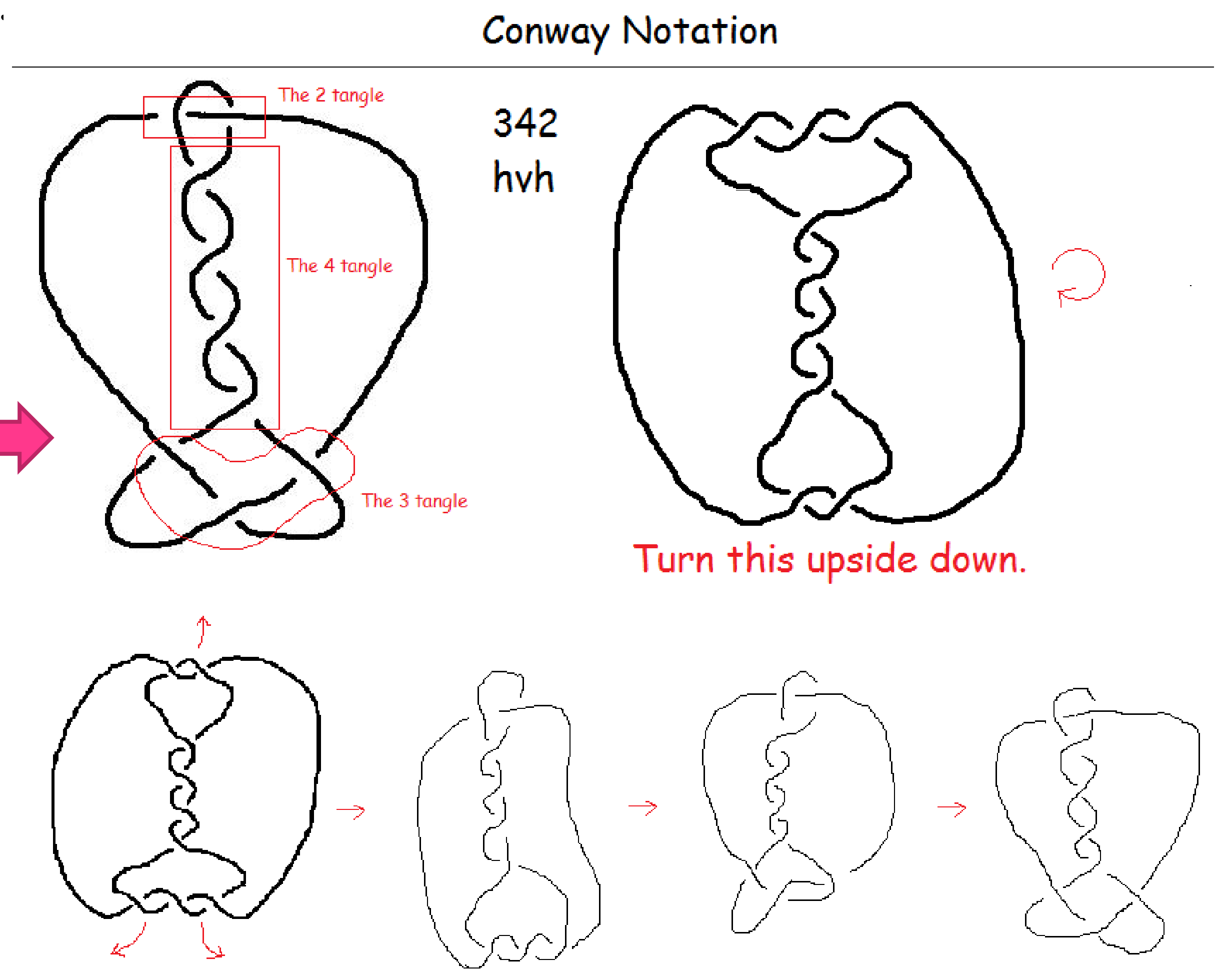
1	3	5	7	9	11	13	15	17
10	16	14	12	18	6	8	4	2

The Dowker Notation for the nine seven is above.

Conway Notation

Conway Notation only applies to rational knots. You put different rational tangles together and close them off to make a rational link that resembles your knot. The number and type of tangles you use is your Conwa

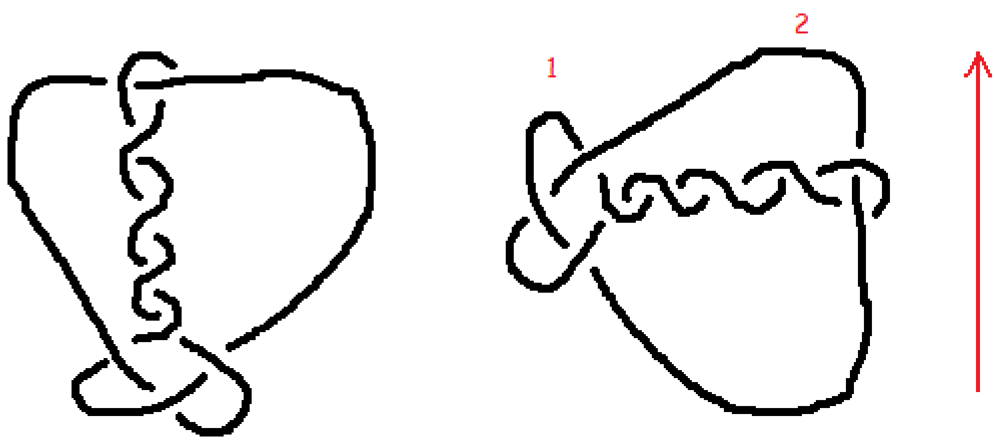
The Conway Notation for the nine seven is 342.



Bridge Number

The bridge number of a knot is the minimum number of maxima (high points) over all projections and all axes.

The bridge number of the nine seven is 2.

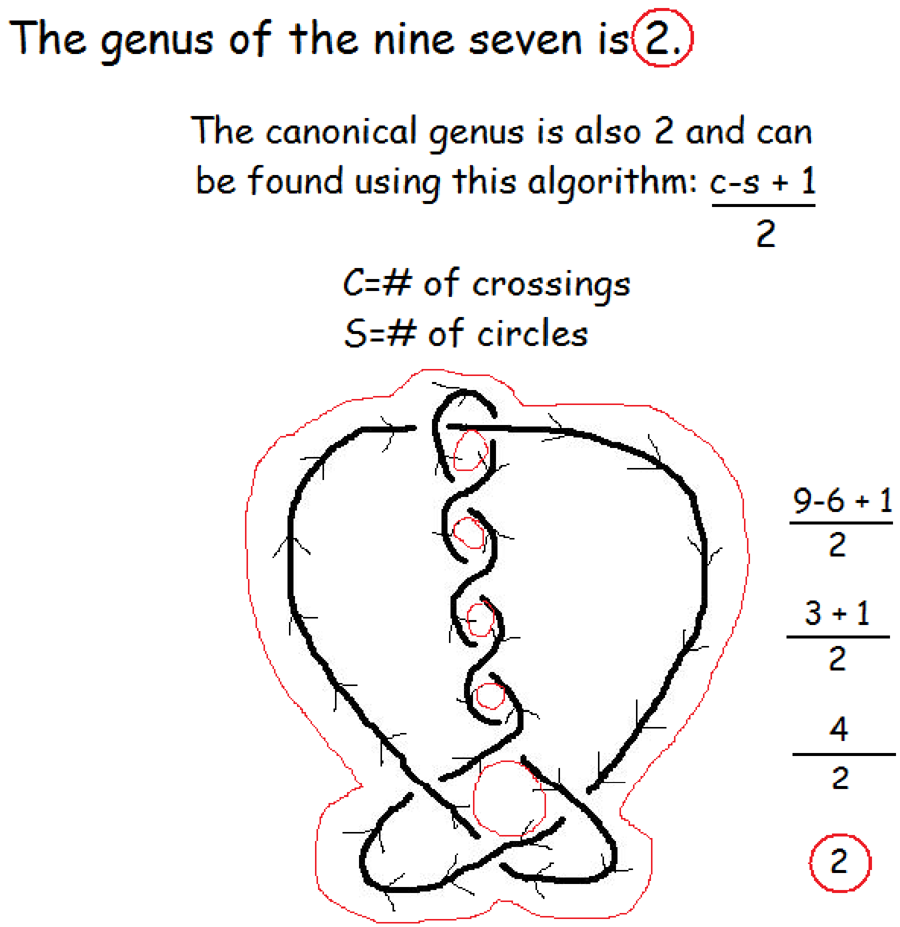


If you turn the nine seven and try with a different axis, then you see that the minimum number of maxima found is 2.

Note: 2-bridge knots are rational.

Genus and Canonical Genus

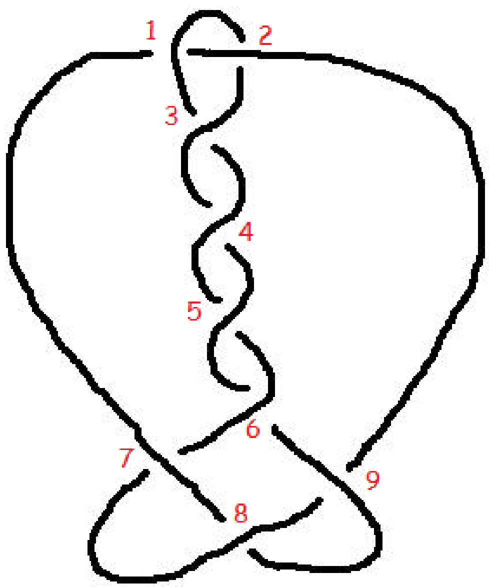
The genus of a knot *k*, g(k), is the minimal possible genus of an orientable surface whose boundary is the knot. The canonical genus of a knot *k*, gc(k), is the minimal possible genus of an orientable surface whose boundary is the knot which can be gotten from Seifert's algorith



CROSSING NUMBER

The crossing number of a knot is the least number of crossings that occur in ANY projection of the knot.

The crossing number of the nine seven is nine. There exists no other projection with fewer crossings.

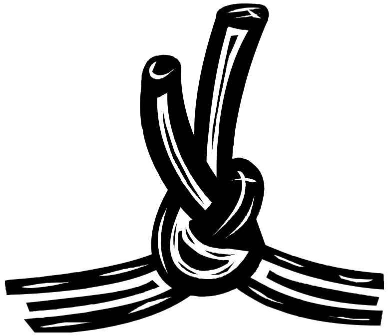


Count the crossings. There are nine in this standard projection, and the fewest number of crossings in any projection of the nine seven will be nine.

Is my knot a Torus Knot?

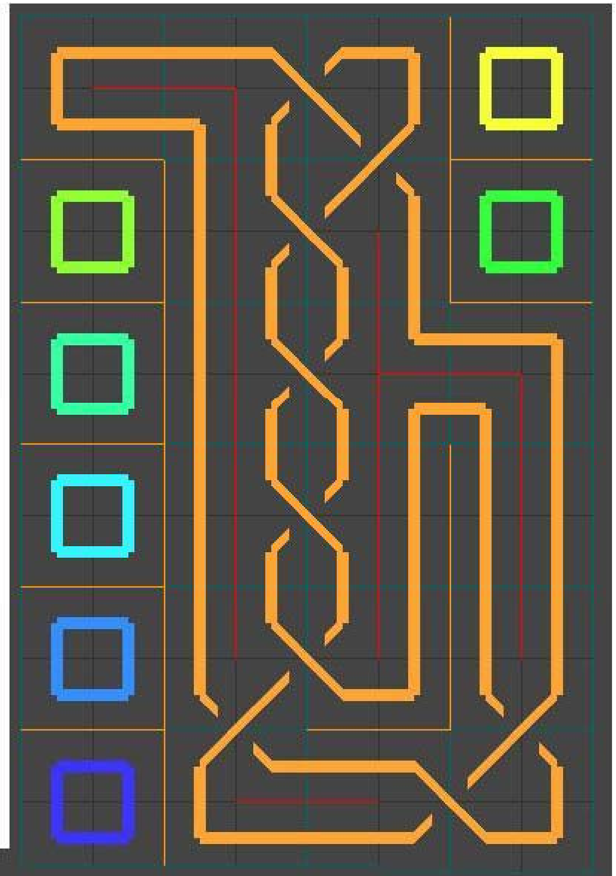
The nine seven cannot be made from links made by torus knots. The nine seven CAN be made from rational links which is why my knot is rational. The nine seven is also rational because it is 2-bridge knot and has a Conway Notation that uses multiplication (342). NOT 3,4,2…

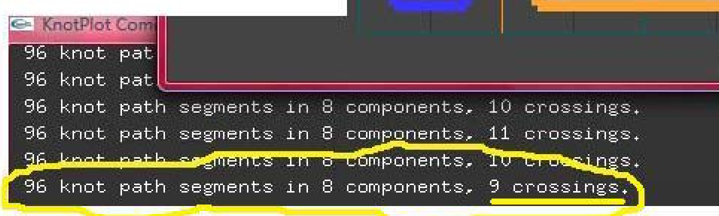
The nine seven is neither a torus nor a satellite knot. Therefore, it must be **hyperbolic.**



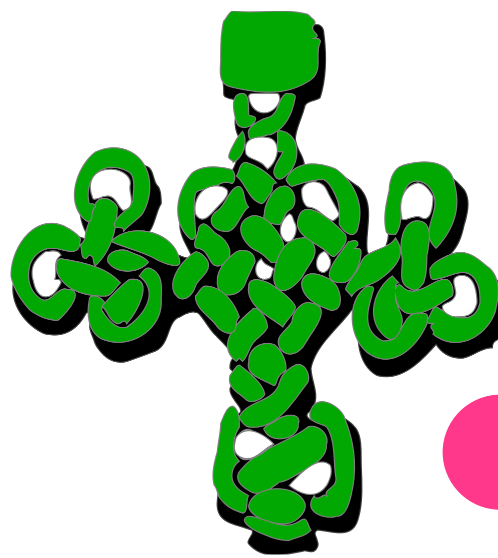
IS THE NINE SEVEN KNOT CELTIC?

<u>My Knot</u> **the 9_7 here is made into a "Celtic" knot using a 4x6 grid.**





YES! The nine seven is Celtic. Here you can see that, using the program Knot Plot, I created the nine seven by using a 4x6 grid. The program did confirm that it is a nine crossing knot.



Alexander Polynomial

The Alexander Polynomial for the nine seven is $3t^{-2} - 7t^{-1} + 9 - 7t + 3t^{2}$