Peter Guthrie Tait

Born: 28 April 1831 in Dalkeith, Midlothian, Scotland
Died: 4 July 1901 in Edinburgh, Scotland

P G Tait's father was John Tait and his mother was Mary Ronaldson. John Tait was a secretary to Walter Francis Scott, the fifth duke of Buccleuch. Peter had two sisters and he began his schooling in the Grammar School in Dalkeith. However, when he was six years old his father died and Peter, with his two sisters and his mother, moved to Edinburgh to live with an uncle John Ronaldson. An Edinburgh banker, John Ronaldson was nevertheless interested in science, in particular in astronomy, geology and with the newly invented photography. He soon interested his young nephew Peter in these subjects and it is fair to say that Peter's interest in science was a direct consequence of his uncle's enthusiasm for the sciences.

When the family moved to Edinburgh Peter, of course, had to leave his school in Dalkeith. He next attended a private school in Circus Place Edinburgh, then in 1841, when he was ten years old, he entered Edinburgh Academy. Lewis Campbell, who later became the professor of Greek at the University of St Andrews, and James Clerk Maxwell were one year above Tait at the Academy. In fact Maxwell was slightly younger than Tait so the difference of one year certainly did not reflect their respective ages.

Tait was top of his class in each one of his six years at Edinburgh Academy. His early interests, however, were not in science but rather in classics. By his fourth year at the Academy mathematics had become his real love and that was the subject in which he really excelled. In 1846 he was placed first in the mathematics section of the Edinburgh Academical Club Prize which was no mean achievement given that he beat Lewis Campbell, who was placed second, and Maxwell who was placed third. In 1847, Tait's final year at Edinburgh Academy, Maxwell had his revenge since he was placed first for the Edinburgh Academical Club Prize with Tait second.

At the age of 16, in November 1847, Tait entered the University of Edinburgh. Maxwell entered Edinburgh University at the same time at Tait and together they attended the second mathematics class taught by Kelland and the natural philosophy (physics) class taught by James David Forbes. Tait remained at Edinburgh University for only one year before entering Peterhouse, Cambridge in 1848. There he was tutored by William Hopkins through what was a remarkable undergraduate career. In January 1852, at the age of twenty, he graduated as senior Wrangler in the Mathematical Tripos. This means that he was placed first among the First Class degrees in mathematics awarded by Cambridge in that year. He was also the first Smith's prizeman.

Maxwell followed Tait to Peterhouse in 1850 but transferred to Trinity where he believed that it was easier to obtain a fellowship. Another fellow student and friend of Tait's was William Steele who was in the same year as Tait and graduated as Second Wrangler. Tait won a Fellowship at Peterhouse and, in addition to coaching undergraduates for the Tripos, he began to collaborate with Steele in writing a text *Dynamics of a particle*. Tragically Steele died before much progress had been made with writing the book but Tait continued with the project and generously published the book under their joint authorship despite having written most of it himself. It was published in 1856.

In September 1854 Tait took up an appointment as professor of mathematics at Queen's College, Belfast. A number of the colleagues and friends he made in Belfast were to have a very significant effect on Tait's career. One of these was Thomas Andrews and the two collaborated in experiments to determine the density of ozone and also the affects of passing electrical discharge through oxygen and other gases. Tait had not been involved in experimental work up to this time and it is certainly due to the influence of Andrews that he added this interest to his growing range of skills. This research carried out with Andrews took Tait towards chemistry and this was a subject he retained an interest in through his career.

Another friendship of real significance was that with Hamilton. Tait had read Hamilton's *Lectures on quaternions* in 1853 while he was still at Cambridge but although the topic fascinated him he was more taken up with physical applications of mathematics at the time and did not pursue the topic at that stage. Then in July 1858 Tait read a paper by Helmholtz in *Crelle's Journal* on the motion of a perfect fluid. Helmholtz's paper *Über Integrale der hydrodynamischen Gleichungen, welche den Wirbelbewegungen entsprechen* began by decomposing the motion of a perfect fluid into translation, rotation and deformation. Tait saw that using quaternions he could express the fluid velocity as a "vector function". It was the physical insight which Hamilton's quaternion differential calculus then gave which impressed Tait and he began to work hard developing a physical theory.

Tait began to correspond with Hamilton in August 1858 and, in reply to Hamilton's question as to how he had stated to work with quaternions, Tait wrote to Hamilton on 7 December 1858 (see for example [6]):-

> *... it was only in August last that I suddenly bethought me of certain formulas I had admired years ago on page* 610 *of your Lectures - and I thought* (*and still think*) *likely to serve my purpose exactly*. (*The matter which more immediately suggested this to me was a paper by Helmholtz in Crelle's Journal* (*Vol. LX*) *which I was reading in July last as soon as we received it ... The title* (*in German*) *I forget - but a manuscript translation of my own which I now have beside me is headed "Vortex motion"* ...).

If Tait's friendship with Hamilton was to prove important for his future research, then other friendships which Tait formed were important in his family life. Two of his friends at Peterhouse were sons of the Rev James Porter and through them Tait met their sister, Margaret Archer Porter, who he married in Belfast on 13 October 1857.

The Chair of Natural Philosophy at the University of Edinburgh became vacant in 1859, J D Forbes having moved to the University of St Andrews. Tait was a candidate for the chair but so was Maxwell who had been forced to seek another post when Marischal College and King's College in Aberdeen combined. Routh, who had been First Wrangler at Cambridge in Maxwell's year, was also a candidate but the real competition was always going to be between Tait and Maxwell. Tait won despite Maxwell's outstanding scientific achievements. When the Edinburgh paper, the *Courant,* reported the result it noted that Tait had been chosen in preference to Maxwell since:-

> *... there is another quality which is desirable in a Professor in a University like ours and that is the power of oral exposition proceeding on the supposition of imperfect knowledge or even total ignorance on the part of pupils.*

The claim that Tait was the better person to teach poorly qualified pupils was certainly a fair one and, of course, Tait's personality meant that he made a stronger impression on the appointing committee rather than the much more reserved Maxwell.

By the time he arrived in Edinburgh in 1860 Tait was making strong contributions in applying Hamilton's quaternions. In the year he took up the chair of Natural Philosophy at Edinburgh he published *Quaternion investigations connected with electro-dynamics and magnetism* in which he reworked Helmholtz's hydrodynamic- electromagnetic analogy in the language of quaternions. As Epple writes in [10]:-

> *... not only quaternion analysis profited from acquiring a new. physical meaning. Quaternionic formulas also helped to grasp physical situations which could be described in terms of fluid motion more easily.*

By 1863 when he published *Note on a quaternion transformation* in the *Proceedings of the Royal Society of Edinburgh,* Tait claimed that:-

> *... the next grand extensions of mathematical physics will, in all likelihood, be furnished by quaternions.*

Hamilton died in 1865 and Tait took over the crusade to give quaternions a leading role in mathematical physics. Among the many contributions he made to the topic we should mention his two important texts *Elementary Treatise on Quaternions* (1867), and *Introduction to Quaternions* (1873). Maxwell was impressed by Tait's many works on physical applications of quaternions and wrote in a letter to William Thomson in 1871:-

> *You should let the world know that the true source of mathematical methods as applicable to physics is to be found in the Proceedings of the Royal Society of Edinburgh. The volume- surface- and line- integrals of vectors and quaternions and their properties as in the course of being worked out by Tait is worth all that is going on in other seats of learning.*

Despite his intense work on quaternions, Tait was involved in many other activities. In 1862 he had published joint work with James A Wanklyn on electricity produced during evaporation and during effervescence. Three years later he published a paper on the motion of iron filings on a vibrating plate which was subjected to a magnetic field. In 1866 he started a joint project with the physicist Balfour Stewart on heating a disk which was rapidly rotating in a vacuum. This was a topic Tait came back to on several occasions throughout his career. Then in 1867 he published, in addition to the treatise on quaternions, his translation of Helmholtz's 1858 article and also the *Treatise on Natural Philosophy* for which he may be best known.

In 1861 Tait had been working on a text on mathematical physics. His friend William Thomson (later Lord Kelvin):-

> *... to my great delight offered to join.*

The two intended to write a two volume work and *Treatise on Natural Philosophy* (1867) was to be the first of the volumes. However the second volume was never written. The treatise, known as 'T & T', was written mainly by Tait who seemed to find time despite his numerous other activities, while Thomson found that his many other activities prevented him finding as much time as Tait to work on the book. Hamilton Dickson writes in [9]:-

> *The work was epoch-making, and created a revolution in scientific development. For the first time 'T & T', as the authors called themselves, traced to Newton the concept of the*

> *'conservation of energy' which was just then obtaining recognition among physicists, and they showed once and for all that 'energy' was the fundamental physical entity and that its 'conservation' was its predominant and all-controlling property.*

We have already detailed major achievements for Tait dated 1867 but there is one further event of that year which we should mention. Helmholtz, in his 1858 paper, described the theoretical behaviour of vortex rings. He claimed that two interacting rings would change size and velocity as they interacted but would retain their ring shape. Tait verified Helmholtz' theoretical claims with experiments with smoke rings in 1867. He used two boxes each with a rubber diaphragm which shot out white smoke rings when the diaphragm was struck. Thomson wrote to Helmholtz on 22 January 1867:-

> *... a few days ago Tait showed me in Edinburgh a magnificent way of producing* [*vortex rings*]. *We sometimes can make one ring shoot through another, illustrating perfectly your description; when one ring passes near another, each is much disturbed, and is seen to be in a state of violent vibration for a few seconds, till it settles again into its circular form. ... The vibrations make a beautiful subject for mathematical work.*

These experiments were to have a major influence on Thomson who saw the permanence of form as a possible explanation for atoms and therefore explain the way that the different elements could be built. Tait was not convinced by Thomson's idea at first, rightly so of course since, although a beautiful idea, it is quite wrong. The idea led Tait, Thomson and Maxwell to begin to work on knot theory since the basic building blocks, in Thomson's vortex atom theory, would be the rings knotted in three dimensions. By Helmholtz' theory of a perfect fluid, these knotted rings, although they could be distorted, would retain the 'same knot' as a circular knotted piece of string that can be moved around yet the form of the knot remains an invariant.

Tait, Thomson and Maxwell exchanged letters in which they invented many topological ideas as they looked at knots. Soon they discovered Listing's 1847 contributions to knot theory. Tait, although at first unconvinced by Thomson's vortex atom theory, began to include the theory in his lecture courses at Edinburgh in the early 1870s and he gave popular lectures describing the theory. In 1876 Tait began an intense study of knots, attempting to classify them. He published seven papers on knots in the *Proceedings of the Royal Society of Edinburgh* in the academic year 1876-77.

Tait considered alternating knots, namely those which when traversing the projection in 2-dimensional space the crossings go alternately over and under. He labelled the n crossings of such a knot A, B, C, ... and then the knot would be described by the sequence of crossings of length $2n$ where each of A, B, C, ... occurred exactly twice when the knot was traversed. There were then two basic problems to solve. Firstly which sequences of the above type corresponded to a knot, and secondly how could it be determined when two knots described by such sequences were the same.

Without any rigorous theory, which would have been well beyond nineteenth century mathematics, Tait began to classify knots using his mathematical and geometrical intuition. By 1877 he had classified all knots with seven crossings but he stopped there. One of the problems he considered after that was the colouring of graphs since he claimed to have a correct proof of the four colour theorem. His proof is fallacious and, sadly, he did not relate colouring of graphs to the knot theory he had considered a few years earlier. Another topic which he had worked on over a number of years was the results of the Challenger expedition on deep sea temperatures. In 1881 Tait published an important paper on the topic in which he showed how to correct the temperature readings because of the high pressures on the thermometers.

He returned to the topic of knots in his address to the Edinburgh Mathematical Society in 1883:-

> *We find that it becomes a mere question of skilled labour to draw all the possible knots having any assigned number of crossings. The requisite labour increases with extreme rapidity as the number of crossings is increased. ... I have not been able to find time to carry out this process further than the knots with seven crossings. ... It is greatly desired that someone, with the requisite leisure, should try to extend this list, if possible up to* 11 ...

Kirkman read the text of Tait's address and began to work on classifying knots with more than seven crossings. He sent Tait his results on knot projections with up to nine crossings in May 1884 but he had not looked at the problem of deciding which of the projections led to equivalent knots. Tait worked on this side of the problem and, considering only alternating knots, solved the equivalence problems within a few weeks. Tait seemed to know how to tell whether two knots were equivalent without rigorous methods. He states this quite clearly in the paper he wrote tabulating the knots where he says that his methods have:-

> *... the disadvantage of being to a greater or less extent tentative. Not that the rules laid down ... leave any room for mere guessing, but they are too complex to be always completely kept in view. Thus we cannot be absolutely certain that by means of such processes we have obtained all the essentially different forms which the definition we employ comprehends.*

Despite the problems Tait knew exactly what he was doing for, remarkably, his tables are correct. When Kirkman sent him all knot projections with 10 crossings in January 1885 again Tait found all in equivalent knots. The tables were printed in September 1885 and again they are completely correct. By then he had received from Kirkman 1581 knot projections with 11 crossings and this time Tait felt that he did not have the time to solve the equivalence problem for these.

It would be quite impossible in an article of this length to cover all the topics which Tait worked on. Knott [6] lists 365 papers and 22 books written by Tait. We will mention two final topics which he worked on after ending his work on knots. Thomson suggested that he work on the kinetic theory of gases and between 1886 and 1892 Tait published more than 20 papers on the topic. In this work he gave what Thomson considered the first proof of the Waterston-Maxwell equipartition theorem.

Tait also wrote a classic paper on the trajectory of golf balls (1896). The subject of golf was one of great interest to Tait. Of his four sons, the third was Frederick Guthrie Tait. He became the leading amateur golfer in 1893 and won the Open Golf Championship in 1896 and again in 1898. Freddie Tait, as he was known in the golfing world, has a street in St Andrews named after him which is not far from my [EFR] home. Freddie was a military man in the Black Watch. He gave up his golf when he volunteered to serve in the Boer War in 1899. He was wounded at Magersfontein on 19 December 1899 and killed during fighting at Koodoosberg on 7 February 1900.

A deeply religious man, Tait wrote, with the physicist Balfour Stewart, *The Unseen Universe* (1875):-

> *... to overthrow materialism by a purely scientific argument.*

Because of the public demand, he wrote a sequel *Paradoxical Philosophy* (1878).

We have painted a very positive picture of Tait in the details we have given above. This is right for he deserves no less. However there was another side to his character which we should mention. He became involved in many arguments with his fellow scientists and at least twice engaged in very public arguments. Tait was prone to let his heart rule his head in such situations and he often came of worse in the scientific debate. One of his disputes was with Heaviside and Gibbs whose vector methods he argued vigorously against over a long period. Certainly Tait came off worst in this arguments, perhaps his heart was too set on quaternionic methods to allow his head to see the importance of the ideas of Heaviside and Gibbs.

Another bitter dispute was with Clausius and Tyndall. Tait was patriotic to the extent that he would let such considerations prejudice his view of science. The dispute began over who was the first to propose the equivalence of work and heat. Tait and Tyndall began an argument over whether Joule or Julius Robert Mayer von Mayer had priority. Tait wrote a highly prejudiced account of the history of thermodynamics which was stupidly pro-British and Tyndall was right to be offended. Then Hopkins stumbled into the controversy when Tyndall had asked him to send him all von Mayer's papers but he was as pro-German as Tait was pro-British when he published an article in 1868 stating that not only did von Mayer have priority but so did the German nation. A more bitter dispute between Tait and Clausius began in 1872 when Maxwell published his *Theory of Heat*. Clausius stated that the British were trying to claim more than they deserved for the theory of heat which, given Tait's writing, was a fair comment. Maxwell, however, had over a number of years fully recognised Clausius's contribution, unlike Tait with his prejudiced approach.

Of course Tait's patriotism also meant that he was a devoted supporter of the Royal Society of Edinburgh which he served faithfully from the time he was elected a Fellow shortly after being appointed to the chair in Edinburgh. He served the Society as General Secretary for 22 years from 1879 until 1901. He won the Gunning Victoria Jubilee Prize and twice the Keith prize from that Society. Although never elected a Fellow of the Royal Society of London, he did have the distinction of receiving that Society's Royal Medal in 1886. Other honours given to Tait included the award of honorary degrees by the University of Glasgow and the University of Ireland, as well as being elected to honorary membership of the academies of Denmark, Holland, Sweden and Ireland.

At a ceremony in Peterhouse to present a portrait of Tait, Lord Kelvin spoke of his friend:-

> *I remember Tait once remarking that nothing but science is worth living for. It was sincerely said, but Tait himself proved it to be not true. Tait was a great reader. He would get Shakespeare, Dickens, and Thackeray off by heart. His memory was wonderful. What he once read sympathetically he ever after remembered. Thus he was always ready with delightful quotations, and these brightened our hours of work. For we did heavy mathematical work, stone breaking was not in it.*

In [3] he is described as follows:-

> *His familiar figure was marked by a certain eccentricity, or carelessness of dress, and some of his intimate friends can scarcely remember ever to have seen him in a dress suit. Dining out was indeed an abomination in his eyes, unless it were informally in the company of one or two kindred spirits.*

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MacTutor History of Mathematics
[http://www-history.mcs.st-andrews.ac.uk/Biographies/Tait.html]