Math 164
Fall, 2009
Quiz 1 - §§1.1, 1.2

Name: **Solutions**
Due Monday, 31 August 2009 at the beginning of class

Complete the following problems. Show all work to receive full credit. You are not allowed to discuss these problems with anyone, or to look for solutions anywhere except in your own head. That means you are not allowed to look at other textbooks or online for solutions. Using any other resources will be considered cheating and will be subject to consequences as set forth in the syllabus.

1. What are the next 3 items in the pattern O, T, T, F, F, S, S, E, N, T, E, ...?

   Solution These are the first letters of the first of the counting numbers: one, two, three, four, five, six, seven, eight, nine, ten, eleven. Therefore the next items in the sequence are the first letters of the words twelve, thirteen, and fourteen. Therefore, they are T, T, F.

2. What are the next 3 numbers in the pattern

   $$1, 10, 1, 20, 1, 30, \ldots?$$

   Solution This pattern alternates between the number 1 and multiples of 10. Therefore, the next three terms are:

   $$1, 40, 1$$

3. Consider the following mathematical illusion: A regular deck of 52 playing cards is shuffled several times by an audience member until everyone agrees that the cards are completely shuffled. Then, without looking at the cards themselves, the magician divides the deck into two equal piles of 26 cards. The magician taps both piles of face-down cards three times. Then, one by one, the magician reveals the cards of both piles. Magically, the magician is able to have the cards arrange themselves so that the number of cards showing black suits in the first pile is identical to the number of cards showing red suits in the second pile. Your challenge is to figure out the secret to this illusion. Explain how this illusion works every time, regardless of the number of red cards in the first pile.

   Solution Each deck of cards has 52 cards: 26 black and 26 red. We divide the deck randomly into two piles of 26. Let's say that the first pile has r_1 red cards and b_1 black cards, and that the second pile has r_2 red cards and b_2 black cards. Since there are 26 cards in each pile, we know that:

   $$r_1 + b_1 = 26 \text{ and}$$
   $$r_2 + b_2 = 26.$$

Since there are 26 red cards and 26 black cards in the deck we also know that

$$r_1 + r_2 = 26 \text{ and}$$

$$b_1 + b_2 = 26.$$

Therefore, using the first equation,

$$r_1 + b_1 = 26$$

gives us

$$r_1 = 26 - b_1$$

. Plugging into the third equation, $r_1 + r_2 = 26$, we get:

$$26 - b_1 + r_2 = 26$$

When we subtract 26 from both sides, we get

$$-b_1 + r_2 = 0$$

which really means

$$b_1 = r_2$$

. In other words, the number of black cards in the second pile is the same as the number of red cards in the first pile. similarly, we can see that the number of red cards in the second pile is the same as the number of black cards in the first pile, as we wanted.

By the way, you do not have to do this algebraically, but do have to have these ideas present in your solution.

I certify that all work contained on this quiz is my own. Sign here: