Math 164
Fall, 2009 Name: **Solutions**
Quiz 1 - §§1.1, 1.2 **Due Monday, 31 August 2009 at the beginning of class**

Complete the following problems. Show all work to receive full credit. You are not allowed to discuss these problems with anyone, or to look for solutions anywhere except in your own head. That means you are not allowed to look at other textbooks or online for solutions. Using any other resources will be considered cheating and will be subject to consequences as set forth in the syllabus.

1. What are the next 3 items in the pattern O, T, T, F, F, S, S, E, N, T, E, ...?

   Solution: These are the first letters of the first of the counting numbers: one, two, three, four, five, six, seven, eight, nine, ten, eleven. Therefore the next items in the sequence are the first letters of the words twelve, thirteen, and fourteen. Therefore, they are T, T, F.

2. What are the next 3 numbers in the pattern

   $$1, 5, 1, 10, 1, 15, \ldots?$$

   Solution: This pattern alternates between the number 1 and multiples of 5. Therefore, the next three terms are:

   $$1, 20, 1$$

3. Some number of coins are spread out on a table. They lie either heads up or tails up. Unfortunately you are blindfolded and thus both the coins and the table upon which they sit are hidden from view. Certainly you can feel your way across the table and count the total number of coins on the table's surface, but you cannot determine if any individual coin rests heads up or down (perhaps you are wearing gloves). You are informed of one fact (beyond the total number of coins on the table): Someone tells you the number of coins that are lying heads up. You can now rearrange the coins, turn any of them over, and move them in any way you wish, as long as the final configuration has all the coins resting (heads or tails up) on the table. Your challenge is to turn over whatever coins you wish and divide the coins into two collections so that one collection of coins contains the same number of heads up coins as the other collection contains. Explain how many coins to turn over, and why your solution is guaranteed to work no matter how many coins are on the table and how many start out heads up.

   Solution: One good way to attack this problem is to think about simple cases to help you see how it should be solved. Remember that, in the end, you have to have a general solution for any number of coins.

Let's start thinking about having two coins on the table. if either 2 are heads up or none are heads up, you can put one coin in each pile and you are done. If you have 1 coin heads up and 1 coin tails up, then you put one coin in each pile and turn over either coin.

If there are three coins on the table and you have all coins tails up, you put one coin in a pile and two coins in the other pile, and you are done. If you have all coins heads up, put one coin in a pile and the two other coins in a pile and flip over one of the coins in the pile of two. The hard part of this problem is what to do if there is one coin heads up or if there are two coins heads up. In these case, you split the coins into a pile with one coin and a pile with two coins. If there was one coin which was heads up, you flip over the coin in the pile of one. If there were two coins which were heads up, you flip over both of the coins in the pile of two coins. This guarantees that there will be the same number of heads in each pile (think about why - what could happen?).

The general solution is the following: If you have n coins on the table, with c coins heads up, split the coins into two piles: one containing c coins and the other containing $n - c$ coins. Then you flip all of the coins in the pile containing c coins. This will guarantee that there will be the same number of heads up coins in both piles (again - this is true, but think about why).

I certify that all work contained on this quiz is my own. Sign here: