Math 143
Fall, 2006
Quiz 8 - §7.2

Name: **Solutions**

Complete the following problems. Show all work to receive full credit.

1. Suppose that the bacteria in a colony can grow unchecked by the law of exponential change. The colony starts with 1 bacterium and doubles every half-hour. How many bacteria will the colony contain at the end of 24 hours?

$$y = y_0 e^{kt}$$
$$2 = 1e^{k \cdot .5}$$
$$\ln 2 = .5k$$
$$k = \frac{\ln 2}{.5} = 2\ln 2 \approx 1.386294361$$

So the model is

$$y = e^{1.3862943361t} \text{ where } t \text{ is measured in hours}$$
$$y = 2.814749767 \cdot 10^{14}$$

Equivalently this can be done the following way:

The population doubles every half-hour means that every hour the population is multiplied by 4. Therefore the exponential equation is:

$$y = 4^t$$

where t is measured in hours.

Then after 24 hours, there are

$$y = 4^{24} \approx 2.814749767 \cdot 10^{14} \text{ bacteria}$$

2. A colony of bacteria is frown under idea conditions in a laboratory so that the population increases exponentially with time. At the end of 3 hours, there are 10,000 bacteria, and at the end of 5 hours there are 40,000. How many bacteria were present initially?

$$y = y_0 e^{kt}$$
$$10,000 = y_0 e^{k \cdot 3} \text{ and } 40,000 = y_0 e^{k \cdot 5}$$
$$\frac{10,000}{e^{3k}} = y_0 \text{ and } \frac{40,000}{e^{5k}} = y_0$$
$$\frac{40,000}{e^{5k}} = \frac{10000}{e^{3k}}$$
$$40,000e^{3k} = 10,000e^{5k}$$

$$4 = e^{2k}$$
$$\ln 4 = 2k$$
$$k = \frac{\ln 4}{2} \approx .6931471806$$

Then

$$10,000 = y_0 e^{3 \cdot 0.6931471806}$$
$$10,000 = 8y_0$$
$$y_0 = 1250 \text{ bacteria}$$