
Complete the following problems by calculating the integrals. Show all work to receive full credit.

1. $\int x2^{(x^2)} dx$

$$\begin{aligned}\text{Let } u &= x^2 \\ du &= 2x dx \\ \frac{1}{2} du &= x dx\end{aligned}$$

$$\begin{aligned}\int x2^{(x^2)} dx &= \int \frac{1}{2}2^u du \\ &= \frac{1}{2} \frac{1}{\ln 2} 2^u + C \\ &= \boxed{\frac{1}{2 \ln 2} 2^{x^2} + C}\end{aligned}$$

2. $\int_1^4 \frac{\ln 2 \log_2 x}{x} dx$

$$\begin{aligned}\int_1^4 \frac{\ln 2 \log_2 x}{x} dx &= \int_1^4 \frac{\ln 2 \frac{\ln x}{\ln 2}}{x} dx \\ &= \int_1^4 \frac{\ln x}{x} dx \\ &= \int_1^4 \ln x \frac{1}{x} dx\end{aligned}$$

$$\begin{aligned}u &= \ln x \\ du &= \frac{1}{x} dx, \text{ so}\end{aligned}$$

$$\begin{aligned}\int_1^4 \ln x \frac{1}{x} dx &= \int_{x=1}^{x=4} u du \\ &= \frac{1}{2} (\ln x)^2 \Big|_1^4 \\ &= \frac{1}{2} (\ln 4)^2 - \frac{1}{2} (\ln 1)^2 \\ &= \boxed{\frac{1}{2} (\ln 4)^2}\end{aligned}$$

$$3. \int \left(\frac{1}{3}\right)^{\tan t} \sec^2 t \, dt$$

$$u = \tan t$$

$$du = \sec^2 t \, dt$$

$$\begin{aligned} \int \left(\frac{1}{3}\right)^{\tan t} \sec^2 t \, dt &= \int \left(\frac{1}{3}\right)^u \, du \\ &= \frac{1}{\ln \frac{1}{3}} \left(\frac{1}{3}\right)^u + C \\ &= \frac{1}{\ln \frac{1}{3}} \left(\frac{1}{3}\right)^{\tan t} + C \\ &= \boxed{\frac{1}{-\ln 3} \left(\frac{1}{3}\right)^{\tan t} + C} \end{aligned}$$