Math 143
Fall, 2005
Quiz 7 - §§5.4, 6.3

Name: **Solutions**

Complete the following problems. Show all work to receive full credit. Solve the following differential equations.

1. $\dfrac{dy}{dx} = \dfrac{e^{2x-y}}{e^{x+y}}$

$$\begin{aligned}
\frac{dy}{dx} &= 2^{2x}e^{-y}e^{-x}e^{-y} \\
\frac{dy}{dx} &= e^{x}e^{-2y} \\
e^{2y}dy &= e^{x}dx \\
\frac{1}{2}e^{2y} &= e^{x} + C \\
e^{2y} &= 2e^{x} + C \\
2y &= \ln\left(2e^{x} + C\right) \\
y &= \frac{1}{2}\ln\left(2e^{x} + C\right)
\end{aligned}$$

2. $\theta\dfrac{dy}{d\theta} - 2y = \theta^3 \sec\theta\tan\theta \qquad y\left(\dfrac{\pi}{3}\right) = 2$

$$\begin{aligned}
\frac{dy}{d\theta} - \frac{2}{\theta}y &= \theta^2\sec\theta\tan\theta \\
\mu(x) = e^{\int -\frac{2}{\theta}d\theta} &= e^{-2\ln\theta} = e^{\ln\theta^{-2}} = \theta^{-2} \\
y &= \theta^2\int \frac{1}{\theta^2}\sec\theta\tan\theta d\theta \\
&= \theta^2\int \sec\theta\tan\theta d\theta \\
&= \theta^2\left(\sec\theta + C\right) \\
y\left(\frac{\pi}{3}\right) &= 2 \\
2 &= \left(\frac{\pi}{3}\right)^2\left(\sec\frac{\pi}{3} + C\right) \\
2 &= \frac{\pi^2}{9}\left(2 + C\right) \\
\frac{18}{\pi^2} &= 2 + C \\
C &= \frac{18}{\pi^2} - 2 \\
y &= \theta^2\left(\sec\theta + \frac{18}{\pi^2} - 2\right)
\end{aligned}$$