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Complete the following problems. Show all work to receive full credit.

1. Find the volume of the solid generated by revolving the region bounded by the  $x$ -axis, the curve  $y = 3x^4$  and the lines  $x = 1$  and  $x = -1$  about

(a) the  $x$ -axis

$$\begin{aligned} V &= \pi \int_{-1}^1 (3x^4)^2 dx \\ &= \pi \int_{-1}^1 9x^8 dx \\ &= \pi x^9 \Big|_{-1}^1 = \pi + \pi = 2\pi \end{aligned}$$

(b) the line  $y$ -axis

$$\begin{aligned} V &= 2\pi \int_0^1 x \cdot 3x^4 dx \\ &= 2\pi \int_0^1 3x^5 dx \\ &= 2\pi \left( \frac{1}{2} x^6 \Big|_0^1 \right) = 2\pi \left( \frac{1}{2} \right) = \pi \end{aligned}$$

2. Find the length of the curve  $y = x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}$  for  $1 \leq x \leq 4$ .

$$\begin{aligned} L &= \int_1^4 \sqrt{\left( \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{\frac{1}{2}} \right)^2 + 1} dx \\ &= \int_1^4 \sqrt{\frac{1}{4}x^{-1} - \frac{1}{2} + \frac{1}{4}x + 1} dx \\ &= \int_1^4 \sqrt{\left( \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{\frac{1}{2}} \right)^2} dx \\ &= \int_1^4 \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{\frac{1}{2}} dx \\ &= \left( x^{\frac{1}{2}} + \frac{1}{3}x^{\frac{3}{2}} \right) \Big|_1^4 \\ &= \frac{10}{3} \end{aligned}$$