Math 143
Fall, 2006
Quiz 5 - §6.3

Name: **Solutions**

Complete the following problems. Show all work to receive full credit.

1. Find the length of the curve $y = \sqrt{1-x^2}$ from $x = -\frac{1}{2}$ to $x = \frac{1}{2}$

$$\frac{dy}{dx} = \frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}} \cdot -2x = -\frac{x}{\sqrt{1-x^2}}$$

$$L = \int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{1 + \left(-\frac{x}{\sqrt{1-x^2}}\right)^2}\, dx$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{1 + \frac{x^2}{1-x^2}}\, dx$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{\frac{1-x^2}{1-x^2} + \frac{x^2}{1-x^2}}\, dx$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{\frac{1}{1-x^2}}dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\, dx$$

$$= \left(\sin^{-1} x\right|_{-\frac{1}{2}}^{\frac{1}{2}} = \sin^{-1}\frac{1}{2} - \sin^{-1}\frac{-1}{2}$$

$$= \frac{\pi}{6} + \frac{\pi}{6} = \boxed{\frac{\pi}{3}}$$

2. Find the length of the curve $x(t) = 8\cos t + 8t\sin t$, $y(t) = 8\sin t - 8t\cos t$ for $0 \le t \le \frac{\pi}{2}$

$$\frac{dx}{dt} = -8\sin t + 8\sin t + 8t\cos t = 8t\cos t$$

$$\frac{dy}{dt} = 8\cos t - 8\cos t + 8t\sin t = 8t\sin t$$

$$L = \int_0^{\frac{\pi}{2}} \sqrt{(8t\cos t)^2 + (8t\sin t)^2}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{64t^2\cos^2 t + 64t^2\sin^2 t}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{64t^2\left(\cos^2 t + \sin^2 t\right)}\, dt = \int_0^{\frac{\pi}{2}} \sqrt{64t^2}\, dt$$

$$= \int_0^{\frac{\pi}{2}} 8t\, dt = 4t^2\Big|_0^{\frac{\pi}{2}}$$

$$= 4\frac{\pi^2}{4} - 2\cdot 0 = \boxed{\pi^2}$$