Math 143
Fall, 2006 Name: **Solutions**
Quiz 4 - §§6.1, 6.2

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Complete the following problems. Show all work to receive full credit.

1. Find the volume of the solid generated by revolving the region in the first quadrant bounded by the curve $x = y - y^3$ about the $x-$axis

   By shells:

$$\begin{aligned} V &= 2\pi \int_0^1 y(y - y^3)\ dy \\ &= 2\pi \int_0^1 y^2 - y^4\ dy \\ &= 2\pi \left( \frac{1}{3}y^3 - \frac{1}{5}y^5 \right|_0^1 \\ &= 2\pi \left( \frac{1}{3} - \frac{1}{5} \right) \\ &= \frac{4\pi}{15} \end{aligned}$$

2. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and $y = \dfrac{x^2}{8}$ about the $y-$axis.

   These functions intersect where

$$\sqrt{x} = \frac{x^2}{8} \Longrightarrow x = \frac{x^4}{64}$$

$$64x = x^4 \Longrightarrow x^4 - 64x = 0$$

$$x(x^3 - 64) = 0 \Longrightarrow x = 0 \text{ or } x = 4$$

   By shells:

$$V = \int_0^4 2\pi x \left( \sqrt{x} - \frac{x^2}{8} \right)\ dx = 2\pi \int_0^4 x^{\frac{3}{2}} - \frac{x^3}{8}\ dx$$

$$= 2\pi \left( \frac{2}{5}x^{\frac{5}{2}} - \frac{1}{32}x^4 \right|_0^4 = 2\pi \left( \frac{64}{5} - 8 \right) = \frac{48\pi}{5}$$

   By washers:

$$V = \int_0^2 \left(y^2\right)^2 - \left(\sqrt{8y}\right)^2\ dy$$