
Complete the following problems. Show all work to receive full credit.

1. Find $\frac{d}{dx} \left(\int_{e^x}^e \ln t \, dt \right)$

$$\begin{aligned} &= -\frac{d}{dx} \int_e^{e^x} \ln t \, dt \\ &= -\ln(e^x) \cdot e^x \\ &= -xe^x \end{aligned}$$

2. Find the following definite integrals:

(a) $\int_0^{\frac{\pi}{2}} \frac{1 + \cos 2t}{2} dt$

$$\begin{aligned} &= \int_0^{\frac{\pi}{2}} \frac{1}{2} + \frac{1}{4} \cos 2t \\ &= \left(\frac{1}{2}t + \frac{1}{4} \sin 2t \right) \Big|_0^{\frac{\pi}{2}} \\ &= \frac{1}{2} \frac{\pi}{2} + \frac{1}{4} \sin \pi - \frac{1}{2} \cdot 0 - \frac{1}{4} \sin 0 \\ &= \frac{\pi}{4} + \frac{1}{4} \cdot 0 \\ &= \frac{\pi}{4} \end{aligned}$$

(b) $\int_0^{\frac{\pi}{4}} e^{\sin x} \cos x \, dx$

$$\begin{aligned} &= e^{\sin x} \Big|_0^{\frac{\pi}{4}} \\ &= e^{\frac{\sqrt{2}}{2}} - e^0 \\ &= e^{\frac{\sqrt{2}}{2}} - 1 \end{aligned}$$

(c) $\int_0^{\sqrt{\ln \pi}} 2xe^{x^2} \cos(e^{x^2}) \, dx$

$$\begin{aligned} &= \sin(e^{x^2}) \Big|_0^{\sqrt{\ln \pi}} \\ &= \sin e^{\ln \pi} - \sin e^0 \\ &= \sin \pi - \sin 1 \\ &= 0 - \sin 1 \\ &= -\sin 1 \end{aligned}$$