Math 143
Fall, 2006
Quiz 3 - §§5.6 6.1

Name: **Solutions**

Complete the following problems. Show all work to receive full credit.

1. Find the area of the region between the curve $y = x^2$ and $y = -2x^4$ from $x = -1$ to $x - 1$.

$$\begin{aligned} A &= \int_{-1}^{1} x^2 - (-2x^4)\ dx \\ &= \int_{-1}^{1} x^2 + 2x^4\ dx \\ &= \frac{1}{3}x^3 + \frac{2}{5}x^5\Big|_{-1}^{1} \\ &= \frac{1}{3} + \frac{2}{5} - \left(-\frac{1}{3} - \frac{2}{5}\right) \\ &= \frac{2}{3} + \frac{4}{5} \\ &= \frac{22}{15} \end{aligned}$$

2. Find the volume of the solid generated by rotating the region between the curve $y = \dfrac{1}{2\sqrt{x}}$, $x = \dfrac{1}{4}$, and $x = 4$ about the $x-$axis.

$$\begin{aligned} V &= \int_{\frac{1}{4}}^{4} \pi \left(\frac{1}{2\sqrt{x}}\right)^2\ dx \\ &= \pi \int_{\frac{1}{4}}^{4} \frac{1}{4x}\ dx \\ &= \pi \left(\frac{1}{4}\ln \mid 4x \mid\right|_{\frac{1}{4}}^{4} \\ &= \pi \left(\frac{1}{4}\ln 16 - \frac{1}{4}\ln 1\right) \\ &= \pi \left(\frac{1}{4}\ln 16 - 0\right) \\ &= \frac{\pi}{4}\ln 16 \end{aligned}$$

3. Find the volume of the solid generated by rotating the region between $y = 4-x^2$ and $y = 2-x$ about the $x-$axis.

These graphs intersect at:

$$4 - x^2 = 2 - x$$
$$0 = x^2 - x - 2$$
$$0 = (x-2)(x+1)$$
$$x = 2 \text{ or } x = -1$$

$$V = \pi \int_{-1}^{2} (4-x^2)^2 - (2-x)^2 \, dx$$
$$= \pi \int_{-1}^{2} 16 - 8x^2 + x^4 - \left(4 - 4x + x^2\right) \, dx$$
$$= \pi \int_{-1}^{2} 16 - 8x^2 + x^4 - 4 + 4x - x^2 \, dx$$
$$= \pi \int_{-1}^{2} 12 - 9x^2 + 4x + x^4 \, dx$$
$$= \pi \left(12x - 3x^3 + 2x^2 + \frac{1}{5}x^5 \right|_{-1}^{2}$$
$$= \pi \left(24 - 24 + 8 + \frac{32}{5} - \left(-12 + 3 + 2 - \frac{1}{5} \right) \right)$$
$$= \pi \left(8 + \frac{32}{5} + 12 - 3 - 2 + \frac{1}{5} \right)$$
$$= \pi \left(15 + \frac{33}{5} \right)$$
$$= \frac{108\pi}{5}$$