Math 143
Fall, 2006 Name: **Solutions**
Quiz 2 - Definite integrals

Complete the following problems. Show all work to receive full credit.

1. Calculate the following integrals:

   (a) $\displaystyle\int_1^4 \frac{1}{t\sqrt{t}}\,dt$

   $$\begin{aligned} &= \int_1^4 \frac{1}{t^{\frac{3}{2}}}\,dt \\ &= \int_1^4 t^{-\frac{3}{2}}\,dt \\ &= -2t^{\frac{-1}{2}}\Big|_1^4 \\ &= \frac{-2}{\sqrt{4}} + \frac{2}{\sqrt{1}} \\ &= -1 + 2 = 1 \end{aligned}$$

   (b) $\displaystyle\int_{-\frac{\pi}{3}}^0 \sec x \tan x\,dx$

   $$\begin{aligned} &= \sec x\big|_{-\frac{\pi}{3}}^0 \\ &= \sec(0) - \sec\left(-\frac{\pi}{3}\right) \\ &= 1 - 2 = -1 \end{aligned}$$

   (c) $\displaystyle\int_0^{\ln 9} e^\theta \left(e^\theta - 1\right)^{\frac{1}{2}}$

   $$\begin{aligned} &= \frac{2}{3}\left(e^\theta - 1\right)^{\frac{3}{2}}\Big|_0^{\ln 9} \\ &= \frac{2}{3}\left(e^{\ln 9} - 1\right)^{\frac{3}{2}} - \frac{2}{3}\left(e^0 - 1\right)^{\frac{3}{2}} \\ &= \frac{2}{3}(9-1)^{\frac{3}{2}} - \frac{2}{3}\cdot 0 \\ &= \frac{2}{3}\cdot 8^{\frac{3}{2}} \end{aligned}$$

(d) $\displaystyle\int_1^3 \frac{(\ln(v+1))^2}{v+1}\,dv$

$$= \int_1^3 (\ln(v+1))^2 \frac{1}{v+1}\,dv$$

$$u = \ln(v+1)$$

$$du = \frac{1}{v+1}dv$$

$$= \int_{v=1}^{v=3} u^2\,du$$

$$= \frac{1}{3}u^3\Big|_{v=1}^{v=3}$$

$$= \frac{1}{3}(\ln(v+1))^3\Big|_1^3$$

$$= \frac{1}{3}(\ln 4)^3 - \frac{1}{3}(\ln 2)^3$$