Math 143
Fall, 2005 Name: **Solutions**
Quiz 20 - §8.7

Complete the following problems. Show all work to receive full credit.

1. Find the Taylor series for $f(x) = \dfrac{x}{1-x}$ around $a = 0$.

$$f'(x) = \frac{(1-x) - x(-1)}{(1-x)^2} = \frac{1-x+x}{(1-x)^2} = \frac{1}{(1-x)^2} = (1-x)^{-2}$$
$$f'(0) = 1$$
$$f''(x) = 2(1-x)^{-3}$$
$$f''(0) = 2$$
$$f'''(x) = 6(1-x)^{-4}$$
$$f'''(0) = 6$$

Therefore the series is

$$0 + x + \frac{2}{2}x^2 + \frac{6}{6}x^3 + \frac{24}{4!}x^4 + \frac{120}{5!}x^5 + \cdots$$
$$= \sum_{n=0}^{\infty} x^{n+1}$$

2. Find the Maclaurin series for $f(x) = \sin x - x + \dfrac{x^3}{3!}$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

So

$$\sin x - x + \frac{x^3}{3!} = \left(\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \right) - x + \frac{x^3}{3!}$$
$$= \sum_{n=2}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$