Math 143
Fall, 2005
Quiz 19 - §8.6

Name: **Solutions**

Complete the following problems. Show all work to receive full credit.

1. Find the values of x for which the series $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$ converges absolutely and conditionally.

   Check absolute convergence using the ratio test:

$$\lim_{n\to\infty} \left| \frac{3^{n+1}x^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n x^n} \right| = \lim_{n\to\infty} \left| \frac{3x}{n+1} \right|$$
$$= 0$$

   Since the limit does not depend upon x, the series converges absolutely for all x.

2. Find the values of x for which the series $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{\sqrt{n^2+3}}$ converges absolutely and conditionally.

   Check absolute convergence using the ratio test:

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1}x^{n+1}}{\sqrt{(n+1)^2+3}} \cdot \frac{\sqrt{n^2+3}}{(-1)^n x^n} \right| = \lim_{n\to\infty} |\, x \,| \frac{\sqrt{n^2+3}}{\sqrt{n^2+2n+4}}$$

$$= \lim_{n\to\infty} \sqrt{\frac{n^2+3}{n^2+2n+4}}\, |\, x \,|$$
$$= |\, x \,|$$

   For this to converge we need

$$|\, x \,| < 1$$

   So $-1 < x < 1$. We need to check endpoints:

   When $x = -1$

$$\sum_{n=0}^{\infty} \frac{(-1)^n(-1)^n}{\sqrt{n^2+3}} = \sum_{n=0}^{\infty} \frac{(-1)^{2n}}{\sqrt{n^2+3}}$$

   This series diverges by the integral test:

Using the trigonometric substitution $n = \sqrt{3}\tan\theta$ and $dn = \sqrt{3}\tan\theta\sec\theta d\theta$

$$\begin{aligned}
\lim_{t\to\infty}\int_0^t \frac{1}{\sqrt{n^2+3}}dn &= \lim_{t\to\infty}\int_{n=0}^t \frac{\sqrt{3}\tan\theta\sec\theta d\theta}{\sqrt{2\tan^2\theta+3}} \\
&= \lim_{t\to\infty}\int_{n=0}^t \frac{\sqrt{3}\tan\theta\sec\theta d\theta}{\sqrt{3}\sec\theta} \\
&= \lim_{t\to\infty}\int_{n=0}^t \tan\theta d\theta \\
&= \lim_{t\to\infty} \ln\mid\sec\theta\mid|_{n=0}^t \\
&= \lim_{t\to\infty} \ln\left|\frac{\sqrt{3}}{\sqrt{n^2+3}}\right|\Bigg|_{n=0}^t \\
&= \lim_{t\to\infty} \ln\left|\frac{\sqrt{3}}{\sqrt{t^2+3}}\right| - \ln\left|\frac{\sqrt{3}}{\sqrt{4}}\right| \\
&= -\infty
\end{aligned}$$

When $x = 1$

$$\sum_{n=0}^{\infty}\frac{(-1)^n 1^n}{\sqrt{n^2+3}}$$

This series converges conditionally by Leibniz.

Therefore the series converges absolutely in the interval $-1 < x < 1$, and conditionally at $x = -1$.