Math 143
Fall, 2005 Name: **Solutions**
Quiz 18 - §8.5

Complete the following problems. Show all work to receive full credit.

1. Does the series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n+1}}$ converge absolutely, converge conditionally or diverge? Explain.

   We will check absolute convergence first:

   $$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$$

   can be examined by the integral test:

   $$\begin{aligned}\int_1^{\infty} \frac{1}{\sqrt{n+1}} &= \lim_{t\to\infty} \int_1^t \frac{1}{\sqrt{n+1}} \\ &= \lim_{t\to\infty} 2\sqrt{n+1}\Big|_1^t \\ &= \lim_{t\to\infty} 2\sqrt{t+1} - 2\sqrt{2} \\ &= \infty\end{aligned}$$

   Therefore the series does not converge absolutely.

   We will check conditional convergence by Leibniz's test:

   The terms $\frac{1}{\sqrt{n+1}}$ are always positive, they are non-increasing, and $\frac{1}{\sqrt{n+1}} \to 0$. Therefore by Leibniz's test, the series converges conditionally.

2. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n+1)^n}{(2n)^n}$ converge absolutely, converge conditionally or diverge? Explain.

   We will test absolute convergence first, so examine $\sum_{n=1}^{\infty} \left(\frac{n+1}{2n}\right)^n$ using the root test:

   $$\lim_{n\to\infty} \sqrt[n]{\left(\frac{n+1}{2n}\right)^n} = \lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} < 1$$

   Therefore the series converges absolutely.

3. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n!)^2}{(2n)!}$ converge absolutely, converge conditionally or diverge? Explain.

We will check absolute convergence by the ratio test:

$$\begin{aligned}
\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| &= \lim_{n\to\infty}\frac{((n+1)!)2}{(2n+2)!}\cdot\frac{(2n)!}{(n!)^2} \\
&= \lim_{n\to\infty}\frac{(n+1)(n+1)}{(2n+2)(2n+1)} \\
&= \lim_{n\to\infty}\frac{(n+1)(n+1)}{2(n+1)(2n+1)} \\
&= \lim_{n\to\infty}\frac{n+1}{2(2n+1)} \\
&= \lim_{n\to\infty}\frac{n+1}{4n+2} \\
&= \frac{1}{4} < 1
\end{aligned}$$

Therefore it converges absolutely.