Math 143
Fall, 2006 Name: **Solutions**
Quiz 17 - §§10.5, 10.6, 10.7

Complete the following problems. Show all work to receive full credit.

1. Convert the polar equation $r = \csc\theta e^{r\cos\theta}$ to a Cartesian equation.

$$\begin{aligned} r &= \csc\theta e^{r\cos\theta} \\ r\frac{1}{\csc\theta} &= e^{r\cos\theta} \\ r\sin\theta &= e^{r\cos\theta} \\ y &= e^x \end{aligned}$$

2. Convert the Cartesian equation $x^2 - y^2 = 1$ to a polar equation.

$$\begin{aligned} (r\cos\theta)^2 - (r\sin\theta)^2 &= 1 \\ r^2\cos^2\theta - r^2\sin^2\theta &= 1 \\ r^2\left(\cos^2\theta - \sin^2\theta\right) &= 1 \\ r^2\cos 2\theta &= 1 \\ r^2 &= \frac{1}{\cos 2\theta} \\ r^2 &= \sec 2\theta \end{aligned}$$

3. Find the length of the curve $r = 1 + \cos\theta \quad 0 \le \theta \le 2\pi$.

$$L = \int_0^{2\pi} \sqrt{(1+\cos\theta)^2 + (-\sin\theta)^2}\ d\theta$$

$$= 2\int_0^{\pi} \sqrt{1 + 2\cos\theta + \cos^2\theta + \sin^2\theta}\ d\theta$$

$$= 2\int_0^{\pi} \sqrt{2 + 2\cos\theta}\ d\theta$$

$$= 2\int_0^{\pi} \sqrt{2(1+\cos\theta)}\ d\theta$$

$$= 2\int_0^{\pi} \sqrt{\frac{4(1+\cos\theta)}{2}}\ d\theta$$

$$= 2\int_0^{\pi} 2\sqrt{\frac{1+\cos\theta}{2}}\ d\theta$$

$$= 2\int_0^{\pi} 2\sqrt{\cos^2\left(\frac{\theta}{2}\right)}\ d\theta$$

$$= 2\int_0^{\pi} 2\cos\frac{\theta}{2}\ d\theta$$

$$= 8\sin\frac{\theta}{2}\Big|_0^{\pi}$$

$$= 8\sin\frac{\pi}{2} - 8\sin 0$$

$$= 8$$