Math 143
Fall, 2005 Name: **Solutions**
Quiz 17 - §8.4

Complete the following problems. Show all work to receive full credit.

1. Does the series $\sum_{n=3}^{\infty} \frac{1}{\ln(\ln n)}$ converge or diverge? Explain.

$$n > \ln n > \ln \ln n$$

Therefore

$$\frac{1}{n} < \frac{1}{\ln n} < \frac{1}{\ln \ln n}$$

The series $\sum \frac{1}{n}$ diverges and $\sum \frac{1}{n} \leq \sum \frac{1}{\ln \ln n}$ so this series diverges, also.

2. Does the series $\sum_{n=1}^{\infty} \frac{4}{\sqrt{n}}$ converge or diverge? Explain.

$$\sum_{n=1}^{\infty} \frac{4}{n^{\frac{1}{2}}}$$

so it diverges by the $p-$series test.

3. Does the series $\sum_{n=1}^{\infty} \frac{1}{n(1+(\ln n)^2)}$ converge or diverge? Explain.

Use the integral test:

$$\lim_{t\to\infty} \int_1^t \frac{1}{x(1+(\ln x)^2)} dx$$

$$= \lim_{t\to\infty} \int_1^t \frac{1}{x} \frac{1}{1+(\ln x)^2} dx$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$= \lim_{t\to\infty} \int_{x=1}^t \frac{1}{1+u^2} du$$

$$= \lim_{t\to\infty} \tan^{-1}(u)\Big|_{x=1}^t$$

$$= \lim_{t\to\infty} \tan^{-1}(\ln x)\Big|_1^t$$

$$= \lim_{t\to\infty} \tan^{-1}(\ln t) - \tan^{-1}(\ln 1)$$

$$= \lim_{t\to\infty} \tan^{-1}(\ln t) - \tan^{-1} 0 = \frac{\pi}{2} - 0$$

which converges, so $\sum_{n=1}^{\infty} \frac{1}{n(1+(\ln n)^2)}$ converges.