
Complete the following problems by solving the following differential equations. Show all work to receive full credit.

1. $(t - 1)^3 \frac{ds}{dt} + 4(t - 1)^2 s = t + 1, \quad t > 1$

$$(t - 1)^3 \frac{ds}{dt} + 4(t - 1)^2 s = t + 1$$

$$\frac{ds}{dt} + \frac{4}{t - 1} s = \frac{t + 1}{(t - 1)^3}$$

$$\mu(t) = e^{\int \frac{4}{t-1} dt}$$

$$= e^{4 \ln(t-1)}$$

$$= e^{\ln(t-1)^4}$$

$$= (t - 1)^4$$

$$\mu(t)s = \int \mu(t)g(t) dt$$

$$(t - 1)^4 \cdot s = \int (t - 1)^4 \frac{t + 1}{(t - 1)^3} dt$$

$$= \int (t - 1)(t + 1) dt$$

$$= \int t^2 - 1 dt$$

$$= \frac{1}{3}t^3 - t + C$$

$$y = \left(\frac{1}{(t - 1)^4} \right) \left(\frac{1}{3}t^3 - t + C \right)$$

$$2. \frac{dy}{dx} + xy = x \quad y(0) = -6$$

$$\begin{aligned}\frac{dy}{dx} + xy &= x \\ \mu(x) &= e^{\int x \, dx} \\ &= e^{\frac{x^2}{2}} \\ \mu(x) y &= \int \mu(x)g(x) \, dx \\ &= \int e^{\frac{x^2}{2}} x \, dx \\ &= e^{\frac{x^2}{2}} + C \\ e^{\frac{x^2}{2}} y &= e^{\frac{x^2}{2}} + C \\ y(0) &= -6 \\ e^{\frac{0^2}{2}} \cdot -6 &= e^{\frac{0^2}{2}} + C \\ -6 &= 1 + C \\ -7 &= C \\ y &= e^{-\frac{x^2}{2}} \left(e^{\frac{x^2}{2}} - 7 \right) \\ &= 1 - 7e^{-\frac{x^2}{2}}\end{aligned}$$

$$3. \sqrt{x} \frac{dy}{dx} = e^{y+\sqrt{x}}$$

$$\begin{aligned}\sqrt{x} \frac{dy}{dx} &= e^{y+\sqrt{x}} \\ \sqrt{x} \frac{dy}{dx} &= e^y e^{\sqrt{x}} \\ e^{-y} dy &= e^{\sqrt{x}} \frac{1}{\sqrt{x}} dx \\ \int e^{-y} dy &= \int e^{\sqrt{x}} \frac{1}{\sqrt{x}} dx \\ -e^{-y} &= 2e^{\sqrt{x}} + C\end{aligned}$$