Math 143
Fall, 2005 Name:
Quiz 16 - §8.3

Complete the following problems. Show all work to receive full credit.

1. Does the series $\sum_{n=0}^{\infty} \frac{2^{n+1}}{5^n}$ converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} \frac{2^{n+1}}{5^n} = \sum_{n=0}^{\infty} 2\left(\frac{2}{5}\right)^n$$
$$= \sum_{n=1}^{\infty} 2\left(\frac{2}{5}\right)^{n-1}$$

Therefore this is a geometric series with $a = 2$ and $r = \frac{2}{5} < 1$. Therefore it converges, and the sum is

$$\sum_{n=1}^{\infty} 2\left(\frac{2}{5}\right)^{n-1} = \frac{2}{1-\frac{2}{5}} = \frac{2}{\frac{3}{5}} = \frac{10}{3}$$

2. Does the series $\sum_{n=0}^{\infty} \left(\frac{e}{\pi}\right)^n$ converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} \left(\frac{e}{\pi}\right)^n = \sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^{n-1}$$

which is a geometric series with $a = 1$ and $r = \frac{e}{\pi} \approx .865 < 1$. Therefore this series converges with sum

$$\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^{n-1} = \frac{1}{1-\frac{e}{\pi}} = \frac{1}{\frac{\pi-e}{\pi}} = \frac{\pi}{\pi-e}$$

3. Does the series $\sum_{n=1}^{\infty} \ln\frac{1}{n}$ converge or diverge? If it converges, find the sum.

Let's examine

$$\lim_{n\to\infty} \ln\frac{1}{n} = \infty$$

Therefore since the sequence doesn't converge to 0, the series diverges.